Strengthening the Rational Closure for Description Logics: an overview

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Motivations

The study of nonmonotonic extensions of DLs is motivated by a very concrete and practical problem in standard ontology languages (specifically, in OWL, Web Ontology Language)

A class inherits "all" the properties of its superclasses

In description logics terminology:
if $C \sqsubseteq D$ (concept C is more specific than D) then C inherits all the properties of D

Example:

NonprofitOrganization $\sqsubseteq$ Organization
Organization $\sqsubseteq$ PaysTaxes
NonprofitOrganization $\sqsubseteq$ $\neg$ PaysTaxes
(nonprofit organizations have an exemption from paying taxes)

Exceptions are very common in knowledge concerning laws and regulations (new laws override old laws) but also in biological and medical ontologies.
Motivations

We want to accommodate such kinds of exceptions in (DL based) ontology languages without sacrificing the logical semantics and exploiting the standard reasoning services offered by DL reasoners (subsumption, instance checking, concept satisfiability, KB consistency), without making complexity explode.

We aim to define a richer language that allows defeasible properties of a class, which may be violated, and a richer logic which solves the conflicts, when possible, without producing the inconsistency of the entire KB.

Of course there are many different alternative ways to do this.
A logic which deals with exception is inherently non-monotonic:

\[
\text{NonprofitOrganization} \sqsubseteq \text{Organization}
\]

\[
\text{Normally an organization pays taxes}
\]

\[
\text{Normally a non-profit organization does not pay taxes}
\]

- from the fact that a ACME is an organization, I would conclude that it pays taxes;
- but coming to know that ACME is a non-profit organization, I would conclude, instead, that it does not pay taxes.

We observe that:

- consequences do not necessarily increase, when the KB is augmented with new axioms (non-monotonicity)
- preference should be given to more specific properties with respect to more general ones (specificity)
Nonmonotonic extensions of DLs

Different proposals have been introduced to extend the basic formalism of Description Logics (DLs) with nonmonotonic reasoning features: to represent *prototypical properties* of concepts, to reason about *defeasible inheritance*, to allow *default negation* as in rule based languages.

- **DLs + (prioritized) defaults** (Baader, Hollunder, 2005, 2005b)
- **DLs + epistemic operators** (Donini, Nardi, Rosati, 2002; Ke, Sattler 2008; Motik, Rosati 2010; Knorr, Hitzler, Maier, 2012)
- **DLs + ASP/WFM** (Eiter, et al., 2004, Eiter, et al., 2011)
- **Datalog +/-** (Cali’, Gottlob, Lukasiewicz, 2009)
- **DLs + circumscription** (Brewka 1987; Cadoli et al., 90; Bonatti, Lutz, Wolter, 2009; Bonatti, Faella, Sauro, 2011)
- **DLs + rational closure** (Casini, Straccia, 2010; Casini, Meyer, Varzinczak, Moodley 2013; Giordano et al., 2013, 2015);
- **DLs + lexicographic and relevant closure** (Casini, Straccia 2012; Casini et al., 2014)
- **$\mathcal{DL}^N$: overriding in DLs** (Bonatti, Petrova, Sauro, 2015, 2018), (Bonatti, Sauro, 2017)
The Description Logic $\mathcal{ALC}$

Language of $\mathcal{ALC}$

Let $N_C, N_R, N_I$ be the set of concept names, role names and individual names.

$\mathcal{ALC}$ concepts:

$$C := A | \top | \bot | \neg C | C \cap C | C \cup C | \forall S.C | \exists S.C$$

where $A \in N_C, R \in N_R$ a first order logic or a polymodal logic (Schild,1991)

A Knowledge Base is a pair $KB = (\text{TBox}, \text{ABox})$:

- TBox contains a finite set of inclusion axioms $C \sqsubseteq D$
  $$\text{Mother\_of\_a\_Doctor} \sqsubseteq \text{Female} \sqcap \exists \text{hasChild}.\text{Doctor}$$

- ABox is a set of individual assertions of the form $C(a)$ and $R(a, b)$, where $a, b \in N_I$, a set of individual names.
  For instance:

  $\text{Female}(\text{mary}), \text{hasFriend}(\text{mary}, \text{carlo})$
  $$(\text{Italian} \sqcap \exists \text{hasFriend}.\text{Ingeneer})(\text{carlo})$$
**ALC Semantics**

An ALC interpretation is any structure \( I = (\Delta^I, \cdot^I) \) where:

- \( \Delta^I \) is a domain;
- \( \cdot^I \) is an interpretation function that maps
  - each concept name \( A \) to set \( A^I \subseteq \Delta^I \),
  - each role name \( R \) to a binary relation \( R^I \subseteq \Delta^I \times \Delta^I \),
  - each individual name \( a \) to an element \( a^I \in \Delta^I \).
- \( \cdot^I \) is extended to complex concepts as follows:
  - \( \top^I = \Delta \), \( \bot^I = \emptyset \), \( (\neg C)^I = \{ x \in \Delta \mid x \notin C^I \} \)
  - \( (C \sqcap D)^I = C^I \cap D^I \), \( (C \sqcup D)^I = C^I \cup D^I \)
  - \( (\exists R.C)^I = \{ x \in \Delta \mid \exists y.(x, y) \in R^I \text{ and } y \in C^I \} \)
  - \( (\forall R.C)^I = \{ x \in \Delta \mid \forall y.(x, y) \in R^I \text{ implies } y \in C^I \} \)

**Satisfiability**

An interpretation \( \mathcal{M} = \langle \Delta, \cdot^I \rangle \) satisfies:

- a concept inclusion axiom \( C \sqsubset D \) if \( C^I \subseteq D^I \);
- an individual assertion \( C(a) \) if \( a^I \in C^I \);
- an individual assertion \( R(a, b) \) if \( (a^I, b^I) \in R^I \).
Specificity

Baader and Hollunder [JAR95] observe that “the question of how to prefer more specific defaults over more general ones [...] is of general interest for default reasoning but is even more important in the terminological case where the emphasis lies on the hierarchical organization of concepts”.

And this principle seems to be convincing also looking at ontologies from a software engineering point of view. Many non-monotonic DLs enforce some form of specificity:

- Prioritized defaults and in prioritized circumscription, the specificity ordering is induced by the hierarchical organization of concepts.
- In the rational closure: the ranking of concepts defines their specificity.
- Refinements of the RC (such as the lexicographic closure) and $\mathcal{DL}^N$: use RC ranking as a measure of specificity.
Preferential extensions of description logics allow defeasible inclusions in the knowledge base to model typical properties of individuals. Kraus Lehmann and Magidor’s conditional assertions $C \triangleright D$ become, for $\mathcal{ALC}$:

- typicality inclusions $T(C) \sqsubseteq D$ (Giordano et al., LPAR 2007, FI 2009) based on the preferential semantics [KLM 90];
- defeasible inclusions $C \bowtie D$ (Britz et al. KR 2008) based on the rational semantics [LM 92].
Description Logic with typicality

Preferential Interpretations
A preferential interpretation is a structure $\langle \Delta, <, \cdot \rangle$ where:

- $\Delta$ and $\cdot$ are a domain and an interpretation function, as in $\mathcal{ALC}$ interpretations;
- $<$ is an irreflexive and transitive relation over $\Delta$ and is well-founded.

**Basic idea**: $x < y$ means: $x$ is more normal than $y$

- $(T(C))^I = \text{Min}_<(C^I)$

Ranked interpretations

modularity: for all $x, y, z \in \Delta$, if $x < y$ then either $x < z$ or $z < y$

Each $x \in \Delta$ has a rank $k_M(x)$, where $k_M : \Delta \to \text{Ord}$

Entailment

- A query $F$ is preferentially (rationally) entailed by $K$ if $\mathcal{M}$ satisfies $F$ in all preferential (ranked) models $\mathcal{M}$ of $K$.
- Linear encoding of a preferential entailment into $\mathcal{ALC}$
A minimal model semantics
As preferential and rational entailment define a weak notion of inference.

Alternative kinds of minimization:

▶ In the logic with typicality, $T(A)$ can be defined as in terms of the Gödel-Löb modality $\square$ as $C \sqcap \Box \neg C$.

$\mathcal{ALC} + T_{\text{min}}$: minimizes the instances of $\neg \Box \neg A$ concepts in preferential models (Giordano et al., AIJ 2013) (related to minimization in circumscriptive KBs (Bonatti et al., 2009));

▶ minimizing the rank of individuals in ranked models related to the Rational Closure [Lehmann and Magidor, 92]) and its semantics.
Rational Closure in DLs

- *Rational Closure (RC)* is an algorithmic construction introduced by Lehmann and Magidor (1992) to get stronger notion of inference with respect to preferential and rational entailment.

- Rational Closure by Lehmann and Magidor has been extended to $\mathcal{ALC}$ in (Casini and Straccia, 2010)

Other formulations of the RC for $\mathcal{ALC}$ in
- (Casini, Meyer, Varzinczak, Moodley, 2013)

and in
- (Giordano, Gliozzi, Olivetti, Pozzato, 2013)
Rational Closure

RC construction assigns a rank to each defeasible inclusion and to each concept: less exceptional concepts have lower rank.

Example

\[\text{Penguin} \sqsubseteq \text{Bird} \quad \text{CartoonPenguin} \sqsubseteq \text{Penguin}\]
\[T(\text{Bird}) \sqsubseteq \text{Fly}\]
\[T(\text{Penguin}) \sqsubseteq \neg \text{Fly}\]
\[T(\text{CartoonPenguin}) \sqsubseteq \text{Fly}\]

\[\begin{array}{ll}
\text{rank 0} & \\
T(\text{Bird}) \sqsubseteq \text{Fly} & \text{rank}(\text{Bird}) = 0 \\
\text{rank 1} & \\
T(\text{Penguin}) \sqsubseteq \neg \text{Fly} & \text{rank}(\text{Penguin}) = 1 \\
\text{rank 2} & \\
T(\text{CartoonPenguin}) \sqsubseteq \text{Fly} & \text{rank}(\text{CartoonPenguin}) = 2
\end{array}\]
Advantages of the rational closure

- The *minimal canonical model semantics* provides a characterization of rational closure for $\mathcal{ALC}$ and $\mathcal{SHIQ}$;
- The rational closure has *good computational properties* (polynomial);
- *can be extended to low complexity DLs* (e.g. $\mathcal{ELO}_\bot$ and $\mathcal{SROEL}$).
- *can be extended to some expressive DLs* (e.g. $\mathcal{SHIQ}$).
Drawbacks of the Rational Closure

- RC disregards defeasible information for quantified concepts. From $T(\text{Student}) \sqsubseteq \text{Young}$ and $(\exists \text{hasFriend.Student})(\text{mary})$ it does not follow: $(\exists \text{hasFriend.Young})(\text{mary})$
  A solution for $\mathcal{EL}_\bot$ (Pensel and Turhan 2018).
- RC does not work for all description logics: the RC of a consistent KB can be inconsistent (partial solutions so far)
- does not provide a tractable approach for ABox minimization
- All or nothing:
  “the blocking of property inheritance problem” [Pearl,90] 
  “the drowning problem” [Benferhat,Dubois,Prade,93].
- Exploits a unique preference relation $<$ among individuals
Drawbacks of the Rational Closure

Rational Closure is too weak: “all or nothing”

If a class is exceptional with respect to some property of a superclass, then it does not inherit any typical property of that superclass.

E.g., as Penguins are exceptional Bird with respect to flying, so they do not inherit any typical property of birds (such as having feather). What we have in RC:

\[
\begin{align*}
Penguin & \sqsubseteq Bird, & \text{Black} & \sqsubseteq \neg\text{Grey} \\
& \text{rank 0} & & \\
\mathbf{T}(Bird) & \sqsubseteq \text{Fly} & \text{Bird} & \approx> \text{Fly}, & \text{HF} \\
\mathbf{T}(Bird) & \sqsubseteq \text{HasFeather} & & \\
& \text{rank 1} & & \\
\mathbf{T}(Penguin) & \sqsubseteq \neg\text{Fly} & \text{Penguin} & \approx> \neg\text{Fly}, \text{Black}, & ? \\
\mathbf{T}(Penguin) & \sqsubseteq \text{Black} & & \\
& \text{rank 2} & & \\
\mathbf{T}(BabyPenguin) & \sqsubseteq \text{Grey} & \text{BabyPenguin} & \approx> ? , \text{Grey}, & ?
\end{align*}
\]
Alternative constructions to Rational Closure:

- The lexicographic closure, originally introduced by Lehmann and, for $ALC$, by (Casini and Straccia, 2012),
- The relevant closure (Casini, Meyer, Moodley, Nortje, 2014)
- Semantics with multiple preferences $<_1, <_2, \ldots$ (Gliozzi, 2016)
- The logic of overriding $DL^N$ (Bonatti, Faella, Petrova, Sauro, 2015)
- Skeptical closure [Pruv, 18]

Multipreferences in other logics

- (Fernandez Gil, 2014) several typicality operators $T_1, T_2, \ldots$ but minimal models as in the logic $ALC + T_{min}$
- (Bonatti Lutz, Wolter., 2009) circumscriptive KBs also allow abnormal instances of a class $C$ with respect to a given aspect $P$ using binary abnormality predicates $Ab(P, x)$. 
The lexicographic closure (Lehmann, 1995) for \(\mathcal{ALC}\) (Casini and Straccia, 2012)

- defines a refinement of rational closure,
- it builds on RC and the notion of specificity: for a concept with rank \(i\) it constructs the bases for \(C\), maximal sets of defeasible inclusions compatible with \(E_i\), giving priority to more specific ones (maximal in the lexicographic order)

\[
\begin{align*}
\text{Penguin} &\sqsubseteq \text{Bird}, \quad \text{Black} \sqcap \text{Grey} \sqsubseteq \bot \\
&\quad \text{rank 0} \\
T(\text{Bird}) &\sqsubseteq \text{Fly} \quad \text{Bird} \approx \succ \text{Fly}, \quad \text{HF} \\
T(\text{Bird}) &\sqsubseteq \text{HasFeather} \\
&\quad \text{rank 1} \\
T(\text{Penguin}) &\sqsubseteq \neg\text{Fly} \quad \text{Penguin} \approx \succ \neg\text{Fly}, \text{Black}, \text{HF} \\
T(\text{Penguin}) &\sqsubseteq \text{Black} \\
&\quad \text{rank 2} \\
T(\text{BabyPenguin}) &\sqsubseteq \text{Grey} \quad \text{BabyPenguin} \approx \succ \neg\text{Fly}, \text{Grey}, \text{HF}
\end{align*}
\]
The lexicographic closure

The lexicographic closure, *uses cardinality* when comparing defaults with the same rank in alternative bases.

\[ \text{WorkingStudent} \sqsubseteq \text{Worker} \sqcap \text{Student} \]

--- rank 0 ---

1. \( T(\text{Worker}) \sqsubseteq \neg \text{Young} \sqcap \neg \text{PT} \quad \text{Worker} \approx \neg \text{Young}, \neg \text{PT} \)
2. \( T(\text{Student}) \sqsubseteq \neg \text{PT} \quad \text{Student} \approx \text{Young}, \neg \text{PT} \)
3. \( T(\text{Student}) \sqsubseteq \text{Young} \)

--- rank 1 ---

4. \( T(\text{WorkingStudent}) \sqsubseteq \text{Graduate\_in\_4\_Years} \)

Two options: \( \langle \emptyset, \{4\}, \{1\} \rangle \) and \( \langle \emptyset, \{4\}, \{2, 3\} \rangle \).

but only *one basis* in the lexicographic closure, the second one:

\( \langle 0, 1, 1 \rangle < \langle 0, 1, 2 \rangle \)

(2 defaults with rank 0 vs. 1) and derives:

\[ T(\text{WorkingStudent}) \sqsubseteq \text{Young} \sqcap \neg \text{PT} \]

*Shall we be more cautious and accept both?*

Multiple inheritance
An alternative: the MP-closure

[Giordano, Gliozzi, arXiv 2018]
Why not comparing the sets of defaults using subset inclusion? ⊂ strict partial order on the Pow(D). Use the natural lexicographic order on tuples of sets (still a strict partial order)

$\langle S_\infty, S_n, \ldots, S_1, S_0 \rangle \prec_{MP} \langle S'_\infty, S'_n, \ldots, S'_1, S'_0 \rangle$.

WorkingStudent $\sqsubseteq$ Worker $\sqcap$ Student

rank 0

(1) $T(Worker) \sqsubseteq \neg Young \sqcap PT$ \hspace{1em} Worker $\approx > \neg Young, PT$
(2) $T(Student) \sqsubseteq \neg PT$ \hspace{1em} Student $\approx > Young, \neg PT$
(3) $T(Student) \sqsubseteq Young$

rank 1

(4) $T(WorkingStudent) \sqsubseteq \text{Graduate}\_in\_4\_Years$

Two bases: $\langle \emptyset, \{4\}, \{1\} \rangle$ and $\langle \emptyset, \{4\}, \{2, 3\} \rangle$ which are not comparable using $\prec_{MP}$. We do not conclude:

$T(WorkingStudent) \sqsubseteq Young \sqcap \neg PT$

We are more cautious, but we lose Rational Monotonicity.
The MP Closure

**Construction:** Take the maximal bases for $C$ according to the lexicographic order $\preceq_{MP}$:

$$\langle S_\infty, S_n, \ldots, S_1, S_0 \rangle \preceq_{MP} \langle S'_\infty, S'_n, \ldots, S'_1, S'_0 \rangle$$

using $\subset$ to compare $S_i$ with $S'_i$, for each $i$.

It is a strict partial order (not necessarily modular).

The **semantics** can be defined as done by Lehmann, but using the lexicographic order $\preceq_{MP}$ to order interpretations:

$$x < y \iff V(x) \preceq_{MP} V(y)$$

$y$ violates defaults more specific (according to the lexicographic ordering $\preceq_{MP}$) than those violated by $x$. **Preferential models.**

In $\mathcal{ALC}$

- map any minimal canonical model $\mathcal{M} = \langle \Delta, <_{RC}, I \rangle$ of the RC to a preferential model $\mathcal{N} = \langle \Delta, <, I \rangle$;
- the MP-closure is a **sound approximation** of the multipreference semantics [Gliozzi 2016];
\( \mathcal{DL}^N \) and the Skeptical Closure (SC)

- A weaker constructions than MP-closure, which compute a single basis, blocking inheritance when there are alternative conflicting defaults with the same rank (multiple inheritance).

\( \text{WorkingStudent} \sqsubseteq \text{Worker} \sqcap \text{Student} \)

\[
\begin{align*}
\text{rank 0} \\
(1) & \quad T(\text{Worker}) \sqsubseteq \text{PayTaxes} \\
\quad & \quad \\text{Worker} \approx > \text{PT} \\
(2) & \quad T(\text{Student}) \sqsubseteq \neg \text{PayTaxes} \\
\quad & \quad \\text{Student} \approx > \text{Young}, \neg \text{PT} \\
(3) & \quad T(\text{Student}) \sqsubseteq \text{Young} \\
\text{rank 1} \\
(4) & \quad T(\text{WorkingStudent}) \sqsubseteq \text{Graduate in 4 Years}
\end{align*}
\]

There are 2 bases including alternative sets of defeasible inclusions with same rank: \( \langle \emptyset, \{4\}, \{1, 3\} \rangle \) and \( \langle \emptyset, \{4\}, \{2, 3\} \rangle \):

No default with rank 0 is accepted! Just take \{4\}. 

No acceptance for Young!

In \( \mathcal{DL}^N \): inconsistent prototype!
Constructions and semantics related to Rational Closure:

- The **lexicographic closure**, originally introduced by Lehmann and, for $\mathcal{ALC}$, by (Casini and Straccia, 2012).
  \[ \text{RC} \prec \text{SC} \prec \text{MP-closure} \prec \text{lexicographic closure} \]

- The **relevant closure** $\prec$ lexicographic closure (Casini, et al., 2014), and **relevant closure** $\prec$ MP-closure (arXiv2019)

- Semantics with **multiple preferences**:
  - (Gliozzi, 2016) preferences associated with aspects
  - (Britz and Varzinczak, 2017) preferences on roles

- The **Logic of overriding $\mathcal{DL^N}$** (Bonatti et al., 2015)

- The **skeptical** and the **MP-closure** (Giordano, Gliozzi, Pruv’18) http://arxiv.org/abs/1807.02879


Conclusions

- DLs is a formidable case study for NMR. No commonly agreed solution so far.
- A major role is played by ASP for low complexity DLs.
- Some classical approach (circumscription) have shown to be robust but have *high complexity* (and also new ones $\mathcal{ALC} + T_{min}$, preferential semantics).
- The rational closure can be computed in a polynomial time. Seems to be a good starting point for the definition of *alternative refinements*.

Still some *problems*:

- for expressive DLs:
  - the RC of a consistent KB is *not always consistent* (consistency of RC can be checked);
  - disregards defeasible information for *existential concepts*;
- does not provide a tractable approach for ABox minimization.
Thank you!!!!!