# Strengthening the Rational Closure for Description Logics: an overview

Laura Giordano and Valentina Gliozzi

DiSIT, University of Piemonte Orientale “Amedeo Avogadro”, Italy
laura.giordano@uniupo.it

Dipartimento di Informatica, Università di Torino, Italy, valentina.gliozzi@unito.it

Trieste 19-21 June 2019

# Motivations

The study of nonmonotonic extensions of DLs is motivated by a very concrete and practical problem in standard ontology languages (specifically, in OWL, Web Ontology Language)

*A class inherits "all" the properties of its superclasses*

## In description logics terminology:

if $C \sqsubseteq D$ ( concept C is more specific than D)
then $C$ inherits all the properties of $D$

## Example:

$NonprofitOrganization \sqsubseteq Organization$
$Organization \sqsubseteq PaysTaxes$
$NonprofitOrganization \sqsubseteq \neg PaysTaxes$
(nonprofit organizations have an exemption from paying taxes)

Exceptions are very common in knowledge concerning laws and regulations (new laws override old laws) but also in biological and medical ontologies.

# Motivations

We want to accommodate such kinds of exceptions in (DL based) ontology languages without sacrificing the *logical semantics* and exploiting the standard *reasoning services offered by DL reasoners* (subsumption, instance checking, concept satisfiability, KB consistency), *without making complexity* explode.

We aim to define a richer language that allows defeasible properties of a class, which may be violated, and a richer logic which solves the conflicts, when possible, without producing the inconsistency of the entire KB.

Of course there are many different alternative ways to do this.

# Exceptions and Non-Monotonic Reasoning

A logic which deals with exception is inherently non-monotonic:

> *NonprofitOrganization* $\sqsubseteq$ *Organization*
> *Normally an organization pays taxes*
> *Normally a non-profit organization does not pay taxes*

- from the fact that a ACME is an organization, I would conclude that it pays taxes;
- but coming to know that ACME is a non-profit organization, I would conclude, instead, that it does not pay taxes.

We observe that:

- consequences do not necessarily increase, when the KB is augmented with new axioms (non-monotonicity)
- preference should be given to more specific properties with respect to more general ones (specificity)

# Nonmonotonic extensions of DLs

Different proposals have been introduced to extend the basic formalism of Description Logics (DLs) with nonmonotonic reasoning features: to represent *prototypical properties* of concepts, to reason about *defeasible inheritance*, to allow *default negation* as in rule based languages.

- DLs + (prioritized) defaults (Baader,Hollunder, 2005, 2005b)
- DLs + epistemic operators (Donini,Nardi,Rosati, 2002; Ke,Sattler 2008; Motik,Rosati 2010; Knorr,Hitzler,Maier, 2012)
- DLs + ASP/WFM (Eiter, et al., 2004, Eiter, et al., 2011)
- Datalog +/- (Calı', Gottlob, Lukasiewicz, 2009)
- DLs + circumscription (Brewka 1987; Cadoli et al., 90; Bonatti,Lutz,Wolter, 2009; Bonatti,Faella,Sauro, 2011)
- preferential DLs (Britz, Heidema, Meyer, 2008; Giordano, Gliozzi, Olivetti, Pozzato, 2007, 2009; Britz, Varzinczak 2017; Pensel,Turhan 2017)
- DLs + rational closure (Casini,Straccia, 2010; Casini,Meyer,Varzinczak,Moodley 2013; Giordano et al., 2013, 2015);
- DLs + lexicographic and relevant closure (Casini,Straccia 2012; Casini et al., 2014)
- $\mathcal{DL}^N$: overriding in DLs (Bonatti, Petrova, Sauro, 2015, 2018), (Bonatti, Sauro, 2017)

# The Description Logic $\mathcal{ALC}$

## Language of $\mathcal{ALC}$

Let $N_C,\ N_R,\ N_I$ be the set of concept names, role names and individual names.

$\mathcal{ALC}$ *concepts*:

$$C := A \mid \top \mid \bot \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall S.C \mid \exists S.C$$

where $A \in N_C$, $R \in N_R$

a first order logic or
a polymodal logic (Schild,1991)

A Knowledge Base is a pair KB = (TBox, ABox):

- TBox contains a finite set of *inclusion axioms* $C \sqsubseteq D$

  $$Mother\_of\_a\_Doctor \sqsubseteq Female \sqcap \exists hasChild.Doctor$$

- ABox is a set of *individual assertions* of the form $C(a)$ and $R(a, b)$, where $a, b \in N_I$, a set of individual names.
  For instance:

  $$Female(mary), \quad hasFriend(mary, carlo)$$
  $$(Italian \sqcap \exists hasFriend.Ingeneer)(carlo)$$

# $\mathcal{ALC}$ Semantics

An $\mathcal{ALC}$ interpretation is any structure $I = (\Delta^I, \cdot^I)$ where:

- $\Delta^I$ is a domain;
- $\cdot^I$ is an interpretation function that maps
  - each concept name $A$ to set $A^I \subseteq \Delta^I$,
  - each role name $R$ to a binary relation $R^I \subseteq \Delta^I \times \Delta^I$,
  - each individual name $a$ to an element $a^I \in \Delta^I$.
- $\cdot^I$ is extended to complex concepts as follows:
  - $\top^I = \Delta$ $\quad\quad$ $\bot^I = \emptyset$ $\quad\quad$ $(\neg C)^I = \{x \in \Delta \mid x \notin C^I\}$
  - $(C \sqcap D)^I = C^I \cap D^I$ $\quad\quad$ $(C \sqcup D)^I = C^I \cup D^I$
  - $(\exists R.C)^I = \{x \in \Delta \mid \exists y.(x, y) \in R^I \text{ and } y \in C^I\}$
  - $(\forall R.C)^I = \{x \in \Delta \mid \forall y.(x, y) \in R^I \text{ implies } y \in C^I\}$

## Satisfiability

An interpretation $\mathcal{M} = \langle \Delta, \cdot^I \rangle$ satisfies:

- a concept inclusion axiom $C \sqsubseteq D$ if $C^I \subseteq D^I$;
- an individual assertion $C(a)$ if $a^I \in C^I$;
- an individual assertion $R(a, b)$ if $(a^I, b^I) \in R^I$

# Specificity

Baader and Hollunder [JAR95] observe that

> "the question of *how to prefer more specific defaults over more general* ones [...] is of general interest for default reasoning but is *even more important in the terminological case where the emphasis lies on the hierarchical organization of concepts*".

And this principle seems to be convincing also looking at ontologies from a software engineering point of view.
Many non-monotonic DLs enforce some form of specificity:

- Prioritized defaults and in prioritized circumscription, the *specificity ordering* is induced by the *hierarchical organization of concepts*.
- In the rational closure: the ranking of concepts defines their specificity.
- Refinements of the RC (such as the lexicographic closure) and $\mathcal{DL}^N$: use RC ranking as a measure of specificity.

# Preferential extensions of DLs

- Preferential extensions of description logics allow defeasible inclusions in the knowledge base to model typical properties of individuals. Kraus Lehmann and Magidor's conditional assertions $C \mid\!\sim D$ become, for $\mathcal{ALC}$:
  - typicality inclusions $\mathbf{T}(C) \sqsubseteq D$ (Giordano et al., LPAR 2007, FI 2009)
    based on the preferential semantics [KLM 90];
  - defeasible inclusions $C \mathrel{\tilde{\sqsubseteq}} D$ (Britz et al. KR 2008) based on the rational semantics [LM 92].

# Description Logic with typicality

## Preferential Interpretations

A preferential interpretation is a structure $\langle \Delta, <, \cdot^I \rangle$ where:

- $\Delta$ and $\cdot^I$ are a domain and an interpretation function, as in $\mathcal{ALC}$ interpretations;
- $<$ is an irreflexive and transitive relation over $\Delta$ and is *well-founded*.
  *Basic idea: $x < y$ means: $x$ is more normal than $y$*
- $(\mathbf{T}(C))^I = Min_<(C^I)$

## Ranked interpretations

*modularity*: for all $x, y, z \in \Delta$, if $x < y$ then either $x < z$ or $z < y$
Each $x \in \Delta$ has a rank $k_{\mathcal{M}}(x)$, where $k_{\mathcal{M}} : \Delta \rightarrow Ord$

## Entailment

- A query *$F$ is preferentially (rationally) entailed by $K$* if $\mathcal{M}$ satisfies $F$ in all preferential (ranked) models $\mathcal{M}$ of $K$.
- *Linear encoding of a preferential entailment into $\mathcal{ALC}$*

## A minimal model semantics

As preferential and rational entailment define a weak notion of inference.

Alternative kinds of minimization:

- In the logic with typicality, $\mathbf{T}(A)$ can be defined as in terms of the Gödel-Löb modality $\Box$ as $C \sqcap \Box\neg C$.
  $\mathcal{ALC} + \mathbf{T}_{min}$: minimizes the instances of $\neg\Box\neg A$ concepts in preferential models (Giordano et al., AIJ 2013) (related to minimization in *circumscriptive KBs* (Bonatti et al., 2009));
- minimizing the rank of individuals in ranked models related to the *Rational Closure* [Lehmann and Magidor, 92]) and its semantics.

# Rational Closure in DLs

- *Rational Closure (RC)* is an algorithmic construction introduced by Lehmann and Magidor (1992) to get stronger notion of inference with respect to preferential and rational entailment.
- Rational Closure by Lehmann and Magidor has been extended to $\mathcal{ALC}$ in (Casini and Straccia, 2010)

  Other formulations of the RC for $\mathcal{ALC}$ in
  (Casini, Meyer, Varzinczak, Moodley, 2013)
  and in
  (Giordano, Gliozzi, Olivetti, Pozzato, 2013)

# Rational Closure

RC construction assigns a rank to each defeasible inclusion and to each concept: less exceptional concepts have lower rank.

## Example

$Penguin \sqsubseteq Bird \qquad CartoonPenguin \sqsubseteq Penguin$
$\mathbf{T}(Bird) \sqsubseteq Fly$
$\mathbf{T}(Penguin) \sqsubseteq \neg Fly$
$\mathbf{T}(CartoonPenguin) \sqsubseteq Fly$

rank 0

$\mathbf{T}(Bird) \sqsubseteq Fly$ $\qquad rank(Bird) = 0$

rank 1

$\mathbf{T}(Penguin) \sqsubseteq \neg Fly$ $\qquad rank(Penguin) = 1$

rank 2

$\mathbf{T}(CartoonPenguin) \sqsubseteq Fly$ $\qquad rank(CartoonPenguin) = 2$

## Advantages of the rational closure

- The *minimal canonical model semantics* provides a characterization of rational closure for $\mathcal{ALC}$ and $\mathcal{SHIQ}$);
- The rational closure has *good computational properties* (polynomial);
- *can be extended to* low complexity DLs (e.g. $\mathcal{ELO}_{\perp}$ and $\mathcal{SROEL}$).
- *can be extended to some expressive DLs* (e.g. $\mathcal{SHIQ}$).

## Drawbacks of the Rational Closure

- RC disregards defeasible information for quantified concepts. From
  $\mathbf{T}(Student) \sqsubseteq Young$ and $(\exists hasFriend.Student)(mary)$
  it does not follow:
  $$(\exists hasFriend.Young)(mary)$$
  A solution for $\mathcal{EL}_{\bot}$ (Pensel and Turhan 2018).
- RC does not work for all description logics: the RC of a consistent KB can be inconsistent (partial solutions so far)
- does not provide a tractable approach for ABox minimization
- All or nothing:
  "the blocking of property inheritance problem" [Pearl,90]
  "the drowning problem" [Benferhat,Dubois,Prade,93].
- Exploits a unique preference relation $<$ among individuals

# Drawbacks of the Rational Closure

Rational Closure is too weak: "all or nothing"

*If a class is exceptional with respect to some property of a superclass, then it does not inherit any typical property of that superclass.*

E.g., as Penguins are exceptional Bird with respect to flying, so they do not inherit any typical property of birds (such as having feather). *What we have in RC*:

$Penguin \sqsubseteq Bird, \quad Black \sqsubseteq \neg Grey$

——— rank 0 ———

$\mathbf{T}(Bird) \sqsubseteq Fly$ $\qquad Bird \approx> Fly, \quad HF$

$\mathbf{T}(Bird) \sqsubseteq HasFeather$

——— rank 1 ———

$\mathbf{T}(Penguin) \sqsubseteq \neg Fly$ $\qquad Penguin \approx> \neg Fly, Black, \ ?$

$\mathbf{T}(Penguin) \sqsubseteq Black$

——— rank 2 ———

$\mathbf{T}(BabyPenguin) \sqsubseteq Grey$ $\qquad BabyPenguin \approx> \ ? \ , Grey, \ ?$

## Alternative constructions to Rational Closure:

- The lexicographic closure, originally introduced by Lehmann and, for $\mathcal{ALC}$, by (Casini and Straccia, 2012),
- The relevant closure (Casini, Meyer, Moodley, Nortje, 2014)
- Semantics with multiple preferences $<_1, <_2, \ldots$: (Gliozzi, 2016)
- The logic of overriding $\mathcal{DL}^N$ (Bonatti, Faella, Petrova, Sauro, 2015)
- Skeptical closure [Pruv, 18]

## Multipreferences in other logics

- (Fernandez Gil, 2014) *several typicality operators* $\mathbf{T}_1, \mathbf{T}_2$,.. but minimal models as in the logic $\mathcal{ALC} + \mathbf{T}_{min}$
- (Bonatti Lutz, Wolter., 2009) circumscriptive KBs also allow *abnormal instances of a class C with respect to a given aspect P* using binary abnormality predicates $Ab(P, x)$.

# The lexicographic closure (Lehmann, 1995) for $\mathcal{ALC}$ (Casini and Straccia, 2012)

- defines a refinement of rational closure,
- it builds on RC and the notion of specificity: for a concept with rank $i$ it constructs the bases for C, maximal sets of defeasible inclusions compatible with $E_i$, giving priority to more specific ones (maximal in the lexicographic order)

$Penguin \sqsubseteq Bird, \quad Black \sqcap Grey \sqsubseteq \bot$

rank 0

$\mathbf{T}(Bird) \sqsubseteq Fly$ $\quad$ $Bird \approx> Fly, \quad HF$

$\mathbf{T}(Bird) \sqsubseteq HasFeather$

rank 1

$\mathbf{T}(Penguin) \sqsubseteq \neg Fly$ $\quad$ $Penguin \approx> \neg Fly, Black, HF$

$\mathbf{T}(Penguin) \sqsubseteq Black$

rank 2

$\mathbf{T}(BabyPenguin) \sqsubseteq Grey$ $\quad$ $BabyPenguin \approx> \neg Fly, Grey, HF$

# The lexicographic closure

The lexicographic closure, *uses cardinality* when comparing defaults with the same rank in alternative bases.

$WorkingStudent \sqsubseteq Worker \sqcap Student$

——— rank 0 ———

(1) $\mathbf{T}(Worker) \sqsubseteq \neg Young \sqcap PT$ $\quad Worker \approx> \neg Young, PT$

(2) $\mathbf{T}(Student) \sqsubseteq \neg PT$ $\quad Student \approx> Young, \neg PT$

(3) $\mathbf{T}(Student) \sqsubseteq Young$

——— rank 1 ———

(4) $\mathbf{T}(WorkingStudent) \sqsubseteq Garduate\_in\_4\_Years$

Two options: $\langle \emptyset, \{4\}, \{1\} \rangle$ and $\langle \emptyset, \{4\}, \{2, 3\} \rangle$.
but only *one basis* in the lexicographic closure, the second one:

$$\langle 0, 1, 1 \rangle \prec \langle 0, 1, 2 \rangle$$

(2 defaults with rank 0 vs. 1) and derives:

$$\mathbf{T}(WorkingStudent) \sqsubseteq Young \sqcap \neg PT$$

*Shall we be more cautious and accept both?*

Multiple inheritance

# An alternative: the MP-closure

[Giordano, Gliozzi, arXiv 2018]

Why not comparing the sets of defaults using subset inclusion?

$\subset$ strict partial order on the Pow(D). Use the natural lexicographic order on tuples of sets (still a strict partial order)

$$\langle S_\infty, S_n, \ldots, S_1, S_0 \rangle \prec_{MP} \langle S'_\infty, S'_n, \ldots, S'_1, S'_0 \rangle.$$

$WorkingStudent \sqsubseteq Worker \sqcap Student$

rank 0

(1) $\mathbf{T}(Worker) \sqsubseteq \neg Young \sqcap PT$ $\quad Worker \approx> \neg Young, PT$

(2) $\mathbf{T}(Student) \sqsubseteq \neg PT$ $\quad Student \approx> Young, \neg PT$

(3) $\mathbf{T}(Student) \sqsubseteq Young$

rank 1

(4) $\mathbf{T}(WorkingStudent) \sqsubseteq Garduate\_in\_4\_Years$

Two bases: $\langle \emptyset, \{4\}, \{1\} \rangle$ and $\langle \emptyset, \{4\}, \{2, 3\} \rangle$ which are not comparable using $\prec_{MP}$. We do not conclude:

$$\mathbf{T}(WorkingStudent) \sqsubseteq Young \sqcap \neg PT$$

We are more cautious, but we loose Rational Monotonicity.

# The MP Closure

*Construction*: Take the maximal bases for $C$ according to the lexicographic order $\prec_{MP}$:

$$\langle S_\infty, S_n, \ldots, S_1, S_0 \rangle \prec_{MP} \langle S'_\infty, S'_n, \ldots, S'_1, S'_0 \rangle$$

using $\subset$ to compare $S_i$ with $S'_i$, for each $i$.

It is a strict partial order (not necessarily modular)

The *semantics* can be defined as done by Lehmann, but using the lexicographic order $\prec^{MP}$ to order interpretations:

$$x < y \textit{ iff } V(x) \prec_{MP} V(y)$$

*y violates defaults more specific* (according to the lexicographic ordering $\prec_{MP}$) *than those violated by x*. *Preferential models*.

In $\mathcal{ALC}$

- map any minimal canonical model $\mathcal{M} = \langle \Delta, <_{RC}, I \rangle$ of the RC to a preferential model $\mathcal{N} = \langle \Delta, <, I \rangle$;
- the MP-closure is a *sound approximation* of the multipreference semantics [Gliozzi 2016];

# $\mathcal{DL}^N$ and the Skeptical Closure (SC)

- A *weaker constructions than MP-closure*, which compute a *single basis*, blocking inheritance when there are alternative conflicting defaults with the same rank (multiple inheritance).

$WorkingStudent \sqsubseteq Worker \sqcap Student$

——— rank 0 ———

(1) $\mathbf{T}(Worker) \sqsubseteq PayTaxes$ $\quad Worker \approx> PT$

(2) $\mathbf{T}(Student) \sqsubseteq \neg PayTaxes$ $\quad Student \approx> Young, \neg PT$

(3) $\mathbf{T}(Student) \sqsubseteq Young$

——— rank 1 ———

(4) $\mathbf{T}(WorkingStudent) \sqsubseteq Garduate\_in\_4\_Years$

There are 2 bases including alternative sets of defeasible inclusions with same rank: $\langle \emptyset, \{4\}, \{1,3\} \rangle$ and $\langle \emptyset, \{4\}, \{2,3\} \rangle$:
No default with rank 0 is accepted! Just take $\{4\}$.
Young not accepted!
In $\mathcal{DL}^N$: inconsistent prototype!

## Constructions and semantics related to Rational Closure:

- The *lexicographic closure*, originally introduced by Lehmann and, for $\mathcal{ALC}$, by (Casini and Straccia, 2012). RC $\prec$ SC $\prec$ MP-closure $\prec$ lexicographic closure
- The *relevant closure* $\prec$ lexicographic closure (Casini, et al., 2014), and *relevant closure* $\prec$ MP-closure (arXiv2019)
- Semantics with *multiple preferences*:
  (Gliozzi, 2016) *preferences associated with aspects*
  (Britz and Varzinczak, 2017) *preferences on roles*
- The *Logic of overriding* $\mathcal{DL}^N$ (Bonatti et al., 2015)
- The *skeptical* and the *MP-closure* (Giordano, Gliozzi, Pruv'18) http://arxiv.org/abs/1807.02879
- *Inheritance-based rational closure* (Casini and Straccia, 2013): a construction combining the rational closure with defeasible inheritance networks.
- (Lukasiewicz 2008) *Probabilistic description logics*.

# Conclusions

- DLs is a formidable case study for NMR.
  No commonly agreed solution so far.
- A major role is played by ASP for low complexity DLs
- Some classical approach (circumscription) have shown to be robust but have *high complexity* (and also new ones $\mathcal{ALC} + \mathbf{T}_{min}$, preferential semantics)
- The rational closure can be computed in a polynomial time. seems to be a good starting point for the definition of *alternative refinements*

  Still some *problems*:
  - for expressive DLs:
    - the RC of a consistent KB is not always consistent (consistency of RC can be checked);
    $\Rightarrow$ a generalized notion of RC in [Bonatti 2019]
    - disregards defeasible information for existential concepts;
  - does not provide a tractable approach for ABox minimization.

Thank you!!!!!