Epistemic Answer Set Programming

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Outline

ASP lacks expressivity [Gelfond 1991]

Example (Gelfond's eligibility program Π_G, ASP-version)

% university rules to decide eligibility for scholarship (X: arbitrary applicant)

$$\mathtt{eligible}(X) \leftarrow \mathtt{highGPA}(X)$$
$$\mathtt{eligible}(X) \leftarrow \mathtt{fairGPA}(X)\,,\ \mathtt{minority}(X)$$
$$\sim\mathtt{eligible}(X) \leftarrow \sim\mathtt{highGPA}(X)\,,\ \sim\mathtt{fairGPA}(X)$$

% disjunctive info: an applicant data for a specific student called Mike

$$\mathtt{highGPA}(\mathit{mike})\ \mathrm{or}\ \mathtt{fairGPA}(\mathit{mike})$$

% if eligibility not determined then interview required (ASP attempt)

$$\mathtt{interview}(X) \leftarrow \mathtt{not}\ \mathtt{eligible}(X), \mathtt{not}\ \sim\mathtt{eligible}(X)$$

Quantification problem in ASP

Example (Mike's eligibility situation, ASP-version)

Π_G :

1. eligible ← highGPA
2. eligible ← fairGPA, minority
3. ∼eligible ← ∼fairGPA, ∼highGPA
4. highGPA or fairGPA ←
5. interview ← not eligible, not ∼eligible

has the following answer sets

$$AS(\Pi_G) = \Big\{ \{\mathtt{highGPA}, \mathtt{eligible}\}, \{\mathtt{fairGPA}, \mathtt{interview}\} \Big\}.$$

⇒ eligible? and ∼eligible? *undetermined*

⇒ interview? *undetermined* too...

Quantification problem in ASP

Example (Mike's eligibility situation, ASP-version)

Π_G :

1. $\mathtt{eligible} \leftarrow \mathtt{highGPA}$
2. $\mathtt{eligible} \leftarrow \mathtt{fairGPA}, \mathtt{minority}$
3. $\sim\mathtt{eligible} \leftarrow \sim\mathtt{fairGPA}, \sim\mathtt{highGPA}$
4. $\mathtt{highGPA}\ \mathrm{or}\ \mathtt{fairGPA} \leftarrow$
5. $\mathtt{interview} \leftarrow \mathtt{not}\ \mathtt{eligible}, \mathtt{not}\ \sim\mathtt{eligible}$

has the following answer sets

$$AS(\Pi_G) = \Big\{ \{\mathtt{highGPA}, \mathtt{eligible}\}, \{\mathtt{fairGPA}, \mathtt{interview}\} \Big\}.$$

$\Rightarrow$ eligible? and $\sim$eligible? *undetermined*

$\Rightarrow$ interview? *undetermined* too...

So epistemic modalities are required in ASP...

Example (Mike's eligibility situation, ASP-version)

Π_G :

1. $\texttt{eligible} \leftarrow \texttt{highGPA}$
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4. $\texttt{highGPA}\ \mathrm{or}\ \texttt{fairGPA} \leftarrow$
5. $\texttt{interview} \leftarrow \mathrm{not}\ \texttt{eligible}, \mathrm{not}\ \sim\texttt{eligible}$

Therefore:

$$\Pi_G \not\models \texttt{eligible}$$

$$\Pi_G \not\models \sim\texttt{eligible}$$

$$\Pi_G \not\models \texttt{interview}$$ (counter-intuitive!)

⇒ **wanted:** quantification over possible answer sets...

Gelfond's solution [Gelfond 1991]

Example (Mike's scholarship eligibility revisited, ES-version)

Π_{KG} :

1. eligible ← highGPA
2. eligible ← minority, fairGPA
3. ~eligible ← ~fairGPA, ~highGPA
4. highGPA or fairGPA ←
5. interview ← not K eligible, not K ~eligible

will have slightly different answer sets

$$AS(\Pi_{KG}) = \Big\{ \{\text{highGPA}, \text{eligible}, \text{interview}\}, \{\text{fairGPA}, \text{interview}\} \Big\}$$

⇒ eligible? and ~eligible? *unknown*

⇒ interview? *YES* (intuitive!)

Gelfond's solution [Gelfond 1991]

Example (Mike's scholarship eligibility revisited, ES-version)

Π_{KG} :

1. $\texttt{eligible} \leftarrow \texttt{highGPA}$
2. $\texttt{eligible} \leftarrow \texttt{minority}, \texttt{fairGPA}$
3. $\sim\texttt{eligible} \leftarrow \sim\texttt{fairGPA}, \sim\texttt{highGPA}$
4. $\texttt{highGPA} \textbf{ or } \texttt{fairGPA} \leftarrow$
5. $\texttt{interview} \leftarrow \textbf{not}\ K\ \texttt{eligible}, \textbf{not}\ K \sim\texttt{eligible}$

will have slightly different answer sets

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$\Rightarrow$ eligible? and $\sim$eligible? *unknown*

$\Rightarrow$ interview? *YES* (intuitive!)

ASP lacks expressivity ctd. [Gelfond 2011]

Example (Closed World Assumption (CWA) , ASP-version)

% p is assumed to be false if there is no evidence to the contrary (ASP attempt)

$$\sim p \leftarrow \texttt{not}\, p$$

Consider: $\Pi = \{p \;\texttt{or}\; q, \sim p \leftarrow \texttt{not}\, p\}$

has the following answer sets

$$AS(\Pi) = \Big\{\{p\}, \{\sim p, q\}\Big\}$$

$\Rightarrow$ p? *unknown*
$\Rightarrow$ but also $\sim p$? *unknown* (counter-intuitive)

upshot: again quantification through answer sets is required....

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upshot: again quantification through answer sets is required....

Two different solutions [Gelfond 2011, Shen et al. 2016]

Example (CWA revisited , ES-version)

% p is assumed to be false if there is no evidence to the contrary (ES attempt)

$\sim p \leftarrow \texttt{not}\, \mathsf{M}\, p$ Gelford's approach [LPNMR, 2011]

$\sim p \leftarrow \texttt{not}\, \mathsf{K}\, p$ Shen and Eiter's approach [AIJ, 2016]

Consider: $\mathsf{K}\,\Pi = \{p \,\texttt{or}\, q, \sim p \leftarrow \texttt{not}\, \mathsf{K}\, p\}$

has the unique answer set

$$AS(\mathsf{K}\,\Pi) = \big\{\{\sim p, q\}\big\} \qquad \text{(now, intuitive!)}$$

$\Rightarrow$ Problem ultimately solved? **NO**, still an open problem...

Two different solutions [Gelfond 2011, Shen et al. 2016]

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Outline

Language of ES ($\mathcal{L}_{ES}$) [Kahl et al., ICLP 2018]

extended the language of ASP by epistemic modalities K and M

idea: quantify over all candidate answer sets and correctly represent *incomplete* information (*non-provability*)

K p – – – p is *known* to be true

M p – – – p may be *believed* to be true

- atoms: (extended) *objective* and *subjective* literals

l	**L**	**g**	**G**
p | $\sim p$	l | not l	K l | M l	g | not g

where p ranges over $\mathbb{P}$.

  - strong negation $\sim$ and default negation (aka, negation as failure) not
  - M l $\stackrel{\text{def}}{=}$ not K not l and M not l $\stackrel{\text{def}}{=}$ not K l (K and M are dual!)

notation:

$\mathbb{O}$-*Lit* — the set of all objective literals

$\mathbb{S}$-*Lit* — the set of all subjective literals

Syntax of ES

rule: a logical statement of the form $\texttt{head} \leftarrow \texttt{body}$

- an *ES rule* $\texttt{r}$ is of the form

$$\texttt{l}_1 \texttt{ or } \ldots \texttt{ or } \texttt{l}_m \leftarrow \texttt{e}_1, \ldots, \texttt{e}_n$$

  - $\texttt{head}(\texttt{r})$: disjunction of objective literals
  - $\texttt{body}(\texttt{r})$: conjunction of arbitrary literals

When $m{=}0$, $\texttt{head}(\texttt{r}) = \bot$ and $\texttt{r}$: *constraint* (headless rule)

- if $\texttt{body}(\texttt{r})$ of a constraint consists solely of extended sub. literals, i.e., $\texttt{G}_1, \ldots, \texttt{G}_n$, then $\texttt{r}$: *subjective constraint*.

When $n{=}0$, $\texttt{body}(\texttt{r}) = \top$ and $\texttt{r}$: *fact*.

program: finite collection of rules

- finite set of ES rules = *epistemic specifications*

Truth conditions of ES

For nonempty $\mathcal{A} \subseteq 2^{\mathbb{O}\text{-}Lit}$, $\mathtt{l} \in \mathbb{O}\text{-}Lit$ and $\mathtt{g} \in \mathbb{S}\text{-}Lit$,

- **truth conditions:**

$$
\begin{array}{lll}
\mathcal{A}, A \models \mathtt{l} & \text{if} & \mathtt{l} \in A; \\
\mathcal{A}, A \models \mathtt{not}\,\mathtt{l} & \text{if} & \mathtt{l} \notin A; \\
\mathcal{A}, A \models \mathtt{K}\,\mathtt{l} & \text{if} & \mathtt{l} \in A \text{ for every } A \in \mathcal{A}; \\
\mathcal{A}, A \models \mathtt{M}\,\mathtt{l} & \text{if} & \mathtt{l} \in A \text{ for some } A \in \mathcal{A}; \\
\mathcal{A}, A \models \mathtt{not}\,\mathtt{g} & \text{if} & \mathcal{A}, A \not\models \mathtt{g}.
\end{array}
$$

- **equivalences:**

$$
\begin{array}{lll}
\mathcal{A} \models \mathtt{M}\,\mathtt{l} & \text{iff} & \mathcal{A} \models \mathtt{not}\,\mathtt{K}\,\mathtt{not}\,\mathtt{l} \\
\mathcal{A} \models \mathtt{not}\,\mathtt{M}\,\mathtt{l} & \text{iff} & \mathcal{A} \models \mathtt{K}\,\mathtt{not}\,\mathtt{l}
\end{array}
$$

$\Rightarrow$ K and M are (1) dual and (2) interchangeable.

Kahl's reduct definition [Kahl, PhD thesis 2014]

Given $\mathcal{A} \subseteq 2^{\mathbb{O}\text{-}Lit}$ and an epistemic logic program (ELP) Π:

- **K-reduct** $\mathtt{r}^{\mathcal{A}}$ of an ES rule $\mathtt{r}$ w.r.t. $\mathcal{A}$

idea: eliminate *K* and *M* (in ASP, we eliminate `not` !)

subjective literal (g)	if *true* in $\mathcal{A}$	if *false* in $\mathcal{A}$
`K l`	replace by `l`	delete rule
`not K l`	remove literal	replace by `not l`
`M l`	remove literal	replace by `not not l`
`not M l`	replace by `not l`	delete rule

$$\Pi^{\mathcal{A}} = \{\mathtt{r}^{\mathcal{A}} \; : \; \mathtt{r} \in \Pi\}$$

remark:
K-reduct is rather complex and lacks an intuitive explanation.

Kahl et al.'s world views (K-WV) [Kahl et al., ICLP 2018]

first define:

$$\mathtt{Ep}(\Pi) = \{\mathtt{not}\, \mathrm{K}\, l \;:\; \mathrm{K}\, l \text{ appears in } \Pi\} \cup \{\mathrm{M}\, l \;:\; \mathrm{M}\, l \text{ appears in } \Pi\}$$

then take its subset w.r.t. $\mathcal{A} \subseteq 2^{\mathbb{O}\text{-}Lit}$

$$\Phi_{\mathcal{A}} = \{\mathtt{G} \in \mathtt{Ep}(\Pi) \;:\; \mathcal{A} \models \mathtt{G}\}$$

finally $\mathcal{A}$ is a *K-world view* (K-WV) of a “constraint-free” Π if:

fixed point property

1. $\mathcal{A} = \mathtt{AS}(\Pi^{\mathcal{A}}) = \{A : A \text{ is an answer set of } \Pi^{\mathcal{A}}\}$

knowledge-minimising property

2. there is no $\mathcal{A}'$ such that $\mathcal{A}' = \mathtt{AS}(\Pi^{\mathcal{A}'})$ and $\Phi_{\mathcal{A}'} \supset \Phi_{\mathcal{A}}$.

Some new language constructs [Kahl et al., ICLP 2018]

effect of a constraint $\mathtt{r}$ over answer sets

- in ASP: (at most) rule it out when it violates $\mathtt{r}$
- in ES: also an additive or subtractive effect (not reqular!)

Solution by Kahl and Leclerc: *world view constraints* (WVCs)

- in the form of subjective constraints
- replace $\leftarrow$ by $\overset{wv}{\leftarrow}$
  - $\overset{wv}{\leftarrow}\varphi$ is read: "it is not a world view if it satisfies φ"

Ex: $\overset{wv}{\leftarrow}\mathtt{not}\,\mathrm{K}\,p$: "it is not a world view if p is not known"
(any world view satisfying $\mathtt{not}\,\mathrm{K}\,p$ should be eliminated)

WVCs can solve the constraint problem?

...**to some extent!** because only works for subjective constraints

- what about for $\leftarrow \mathrm{K} p \,,\; q$?

Definition (Kahl and Leclerc's restricted solution)

Let Π be an ELP containing WVCs such that $\Pi = \Pi_0 \cup \Pi_{wvc}$

- Π_{wvc}: set of all WVCs occurring in Π
- Π_0 is a constraint-free part of Π.

Then $\mathcal{A}$ is a K-WV of Π if $\mathcal{A}$ is a K-WV of Π_0 and $\mathcal{A}$ does not violate any constraint in Π_{wvc}.

Outline

Shen and Eiter's (SE's) reduct definition [SE, AIJ 2016]

SE use $\mathtt{not}\,\mathsf{K}$ (*epistemic negation*) to minimise knowledge

- first remember:

$$\mathtt{Ep}(\Pi) = \{\mathtt{not}\,\mathsf{K}\,l \;:\; \mathsf{K}\,l \text{ appears in } \Pi\} \cup \{\mathsf{M}\,l \;:\; \mathsf{M}\,l \text{ appears in } \Pi\}$$

- then take its subset $\Phi \subseteq \mathtt{Ep}(\Pi)$ (they call it a *guess*)
- and take SE-reduct $\mathtt{r}^{\Phi}$ for each $\mathtt{r} \in \Pi$: given $\mathcal{A} \subseteq 2^{\mathbb{O}\text{-}Lit}$,
  - **SE-reduct** $\mathtt{r}^{\Phi}$ of an ES rule $\mathtt{r}$ w.r.t. Φ

idea: eliminate *K* and *M* (aligning with K-reduct)

epistemic-negated sub. literal ($\mathtt{G}$)	if $\mathtt{G} \in \Phi$	if $\mathtt{G} \in \mathtt{Ep}\Pi \setminus \Phi$
$\mathtt{not}\,\mathsf{K}\,\mathtt{l}$	replace by $\top$	replace by $\mathtt{not}\,\mathtt{l}$
$\mathsf{M}\,\mathtt{l}$	replace by $\top$	replace by $\mathtt{not}\,\mathtt{not}\,\mathtt{l}$

- next form

$$\Pi^{\mathcal{A}} = \{\mathtt{r}^{\mathcal{A}} \;:\; \mathtt{r} \in \Pi\}$$

- finally consider $\Phi_{\mathcal{A}} = \{\mathtt{G} \in \mathtt{Ep}(\Pi) \;:\; \mathcal{A} \models \mathtt{G}\}$

SE's world views (SE-WV) [SE, AIJ 2016]

$\mathcal{A}$ is a *SE-world view* (SE-WV) of a "constraint-free" Π if:

fixed point property

1. $\mathcal{A} = \mathtt{AS}(\Pi^{\Phi}) = \{A \ : \ A \text{ is an answer set of } \Pi^{\Phi}\}$;

2. $\Phi_{\mathcal{A}}$ agrees with Φ, i.e., $\Phi_{\mathcal{A}} = \Phi$;

knowledge-minimising property

3. Φ is maximal, i.e., there is no bigger guess $\Phi' \supset \Phi$ such that $\mathcal{A}' = \mathtt{AS}(\Pi^{\Phi'})$ and $\Phi_{\mathcal{A}'} = \Phi'$ for some nonempty collection $\mathcal{A}'$ of consistent sets of objective literals.

⇒ but SE-WVs cannot handle ES including constraints either...

Outline

Epistemic equilibrium logic (EEL) [FHS, IJCAI 2015]

Equilibrium logic (EL):

- today's general purpose nonmonotonic reasoning
- first proposed by Pearce [1996] as a logical foundation of *answer set programming* (ASP) [Lifschitz,Gelfond,. . .]
- based on *here-and-there logic* (HT):
  - a well-known nonclassical monotonic logic [Heyting,Gödel]

Epistemic equilibrium logic (EEL) in a nutshell:

- extend EL with (nondual) epistemic modal operators K and $\hat{K}$
  - semantics via EEMs (minimise truth just as in EL)
  - EEMs are not strong enough as a new semantics for ES
- so, FHS define a selection process over EEMs
- and propose AEEMs (min. knowledge/max. ignorance)
  - inspired by Moore's "Autoepistemic logic" [1980] and Levesque's "All-that-I-know logic" [1990]

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Equilibrium logic (EL):

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Advantages and disadvantages of EEL

Good points:

- The AEEM approach can handle a more general language
- EEMs minimise truth in a standard way (aligning with EMs)

Problems:

- AEEMs depend on 2 different orderings
  - set inclusion $\subseteq$ and a preference ordering $\leq_\varphi$
    - the latter is a bit complex
    - no order between these orderings
    - fortunately not any clash has been found so far
- cannot cope with ES including constraints

Outline

Novelty offered by Epistemic ASP (EASP)

- EASP allows only K, but K may also appear in head
- `not` can only appear in front of literals and not included in a literal formation
- our semantics enjoys a 2-fold computation procedure
- our reduct definition is oriented to eliminate `not`
- our approach can handle programs with constraints

Our aim and motivation

aim:

- solve the problem with ELPs including constraints
- propose a more natural generalisation of ASP

motivating example:

epis. spec. Π	K-WVs	SE-WVs	AEEMs
Π_1: $p \leftarrow \texttt{not}\, q$ $q \leftarrow \texttt{not}\, p$ $r \vee s \leftarrow \texttt{not}\, \mathrm{K}\, p$	$\{\{p, r\}, \{q, r\}, \{p, s\}, \{q, s\}\}$	*same*	*same*
Π_2: $p \leftarrow \texttt{not}\, q$ $q \leftarrow \texttt{not}\, p$ $r \vee s \leftarrow \texttt{not}\, \mathrm{K}\, p$ $\leftarrow r$ $\leftarrow s$	$\{\{p\}\}$	$\{\{p\}\}$	$\{\{p\}\}$

What would we expect? no world views/AEEMs (intuitive!)

Language of EASP ($\mathcal{L}_{\text{EASP}}$)

extended the language of ASP with epistemic modal operator K

- literals (λ) :*objective* literals ($\mathtt{l}$) and *subjective* literals ($\mathtt{g}$)

$\mathtt{l}$	$\mathtt{g}$
$p \mid \sim p$	$\mathrm{K}\, p \mid \mathrm{K} \sim p$

where p ranges over $\mathbb{P}$.

- EASP rules are of the following form:

$$\lambda_1 \;\mathtt{or}\; \ldots \;\mathtt{or}\; \lambda_k \leftarrow \lambda_{k+1}\,,\; \ldots\,,\; \lambda_m\,, \mathtt{not}\; \lambda_{m+1}\,,\; \ldots\,,\; \mathtt{not}\; \lambda_n$$

  - *positive* rules — without negation as failure (NAF) $\mathtt{not}$

- (pos.) EASP program: finite collection of (pos.) EASP rules

⇒ **ASP**: EASP in which literals are restricted to objective literals

Constraint-free and positive EASP programs

semantics: via *epistemic stable models* (ESMs)

Definition (weakening of a point in an S5 model $\mathcal{A} \subseteq 2^{\mathbb{O}\text{-}Lit}$)

Given a (subset) map $\mathsf{s} : \mathcal{A} \to 2^{\mathbb{O}\text{-}Lit}$ such that $\mathsf{s}(A) \subseteq A$ for every $A \in \mathcal{A}$, $\mathsf{s} \neq id$ on $\mathcal{A}$ and $\mathsf{s}|_{\mathcal{A}\setminus\{A\}} = id$,
$\langle \mathsf{s}[\mathcal{A}], \mathsf{s}(A)\rangle$: weakening of $\mathcal{A}$ at a point $A \in \mathcal{A}$.
notation: $\langle \mathsf{s}[\mathcal{A}], \mathsf{s}(A)\rangle \lhd \langle \mathcal{A}, A\rangle$.

Ex: $\left\{\underline{\emptyset}, \{q,r\}\right\} \lhd \left\{\underline{\{p\}}, \{q,r\}\right\}$.

Definition (nonmono. satisfaction reln $\models^*$ minimising truth)

Given a pointed S5 model $\langle \mathcal{A}, A\rangle$ and an EASP program Π,
$\mathcal{A}, A \models^* \Pi$ iff

1. $\mathcal{A}, A \models \Pi$ and
2. $\mathsf{s}[\mathcal{A}], \mathsf{s}(A) \not\models \Pi$ for every map s viz. $\langle \mathsf{s}[\mathcal{A}], \mathsf{s}(A)\rangle \lhd \langle \mathcal{A}, A\rangle$.

Ex: $\left\{\{p\}, \underline{\{r\}}\right\} \models^* q \,\texttt{or}\, r$.

Definition (generalisation of answer set defn in ASP to EASP)

$\mathcal{A}$ is a *minimal model* of Π if $\mathcal{A}, A \models^* \Pi$ for every $A \in \mathcal{A}$.

Example

Consider the following (pos. & constraint-free) EASP program Σ:

$$p \;\texttt{or}\; q \leftarrow$$
$$s \leftarrow q$$
$$r \leftarrow \mathsf{K}\, p$$

Claim: $\big\{\{p\},\{q,s\}\big\}$ is a minimal model of Σ: indeed,

- $\big\{\underline{\{p\}},\{q,s\}\big\} \models \Sigma$ while its only weakening $\big\{\underline{\emptyset},\{q,s\}\big\} \not\models \Sigma$.
- $\big\{\{p\},\underline{\{q,s\}}\big\} \models \Sigma$ while all its weakenings, i.e, $\big\{\{p\},\underline{\{q\}}\big\}$, $\big\{\{p\},\underline{\{s\}}\big\}$ and $\big\{\{p\},\underline{\emptyset}\big\}$ do not satisfy it.

$\{\{p,r\}\}$ and $\{\{q,s\}\}$ are the other (unintended) minimal models of Σ.

$\Rightarrow$ minimality of truth does not guarantee intuitive results.

Definition (epistemic stable model)

Let $\mathcal{A}$ be a nonempty collection of consistent sets of objective literals. Then $\mathcal{A}$ is an *epistemic stable model* (ESM) of a constraint-free and positive EASP program Π if

truth-minimising condition

1. $\mathcal{A}$ is a minimal model of Π;

knowledge-minimising condition

2. there is no minimal model $\mathcal{A}'$ of Π such that $\Phi_{\mathcal{A}} \subset \Phi_{\mathcal{A}'}$;
(it means that $\mathcal{A}$ makes max. possible sub. literals in Π false)
3. there is no minimal model $\mathcal{A}'$ of Π such that $\mathcal{A} \subset \mathcal{A}'$.

$\Rightarrow$ however, 2nd may not be the right approach (but it works) Why?
In ASP, we do not follow such a procedure with `not l`...

- Consider p `or` `not` p in ASP!
  - $\emptyset$ and $\{p\}$ are the answer sets, but we do not choose $\emptyset$.
- Consider also $\mathrm{K}\,p$ `or` `not` $\mathrm{K}\,p$!

Constraint-free, positive EASP programs ctd.

Example

Consider the following EASP program Σ once again:

$$p \,\texttt{or}\, q \leftarrow$$
$$s \leftarrow q$$
$$r \leftarrow \mathsf{K}\, p$$

Σ has 3 min. models: $\mathcal{A}_1{=}\{\{p\},\{q,s\}\}$, $\mathcal{A}_2{=}\{\{p,r\}\}$ & $\mathcal{A}_3{=}\{\{q,s\}\}$.

- $\Phi_{\mathcal{A}_1} = \Phi_{\mathcal{A}_3} = \{\mathsf{K}\, p\} \supset \Phi_{\mathcal{A}_2} = \emptyset$.
- $\mathcal{A}_1 \supset \mathcal{A}_3$.

$\therefore \texttt{ESM}(\Sigma) = \{\mathcal{A}_1\}$.

What if Π contains constraints?

- first, we separate Π into 2 disjoint subprograms:
  - $\mathbb{T}(\Pi) = \{\mathtt{r} \in \Pi \ : \ \mathtt{r} \text{ is a constraint}\}$ ("top")
    - where we evaluate candidate ESMs.
  - $\mathbb{B}(\Pi) = \Pi \setminus \mathbb{T}(\Pi)$ — constraint-free (main) part of Π ("bottom")
    - where we decide candidate ESMs.

$\Rightarrow$ we ensure constraints to behave regularly as in ASP!

- then, we compute $\mathtt{ESM}(\mathbb{B}(\Pi))$
- finally, we evaluate each $\mathcal{A} \in \mathtt{ESM}(\mathbb{B}(\Pi))$ wrt. their behaviour on $\mathbb{T}(\Pi)$: *accept*, *refute* or *reorganise*!

How do we evaluate candidate ESMs on $\mathbb{T}(\Pi)$?

Let $\varphi = \bigvee_{\mathtt{r} \in \mathbb{T}(\Pi)} \mathtt{body}(\mathtt{r})$.
Then, for every $\mathcal{A} \in \mathtt{ESM}(\mathbb{B}(\Pi))$ and every $A \in \mathcal{A}$,

- if $\mathcal{A}, A \not\models \varphi$, then we *accept* $\mathcal{A}$ and call it $\mathcal{A}_{accept}$;
- if $\mathcal{A}, A \models \varphi$, then we *eliminate* $\mathcal{A}$.
- Finally, we *reorganise* the rest in such a way that we take the biggest possible subset $\mathcal{A}_{new} \subseteq \mathcal{A}$ viz.
  - $\mathcal{A}_{new}$ is still a minimal model of $\mathbb{B}(\Pi)$ and
  - $\mathcal{A}_{new}, A \not\models \varphi$ for every $A \in \mathcal{A}_{new}$.

  In other words, $\mathcal{A}_{new}$ turns into $\mathcal{A}_{accept}$.

remark: If $\mathbb{T}(\Pi)$ only contains sub. constraints then we either refute or accept the ESMs of $\mathbb{B}(\Pi)$.

Let's turn back our example!

Example

Consider the following EASP program Σ once again:

$$p \,\texttt{or}\, q \leftarrow$$
$$s \leftarrow q$$
$$r \leftarrow \mathrm{K}\, p$$

remember: $\mathcal{A}_1 = \{\{p\}, \{q, s\}\}$ is the unique ESM of Σ.

- Take $\Sigma_0 = \{\leftarrow r\}$: $\{\underline{\{p\}}, \{q, s\}\} \not\models r$ and $\{\{p\}, \underline{\{q, s\}}\} \not\models r$. So, $\mathcal{A}_1$ passes our test (accept!). $\therefore$ $\texttt{ESM}(\Sigma \cup \Sigma_0) = \mathcal{A}_1$.
- Take $\Sigma_1 = \{\leftarrow \texttt{not}\, \mathrm{K}\, q\}$: $\mathcal{A}_1 \models \texttt{not}\, \mathrm{K}\, q$ So, $\mathcal{A}_1$ fails to be an ESM of $\Sigma \cup \Sigma_1$ (refute!). $\therefore$ $\texttt{ESM}(\Sigma \cup \Sigma_1) = \emptyset$.
- Take $\Sigma_2 = \{\leftarrow p\}$. Since $\{\underline{\{p\}}, \{q, s\}\} \models p$, we delete $\{p\}$ from $\mathcal{A}_1$ and result in $\mathcal{A}_{new} = \{\{q, s\}\}$. $\mathcal{A}_{new}$ is still a minimal model of Σ, so we accept it (reorganise!). $\therefore$ $\texttt{ESM}(\Sigma \cup \Sigma_2) = \{\{q, s\}\}$.

What if Π is not positive?

then we first take the reduct!

- our reduct defn is oriented to eliminate NAF only as in ASP!

Definition (generalisation of the reduct definition of ASP)

Let Π be a "constraint-free" EASP program.
Let $\mathcal{A} \subseteq 2^{\mathbb{O}\text{-}Lit}$ be nonempty and $A \in \mathcal{A}$.Then,

- the *reduct* $\Pi^{\langle \mathcal{A}, A \rangle}$ of Π w.r.t. $\langle \mathcal{A}, A \rangle$ is given by replacing every occurrence of $\texttt{not}\,\lambda$ with
  - $\bot$ if $\mathcal{A}, A \models \lambda$ (for $\lambda = \mathtt{l}$ if $A \models \mathtt{l}$; for $\lambda = \mathsf{K}\,\mathtt{l}$ if $\mathcal{A} \models \mathsf{K}\,\mathtt{l}$);
  - $\top$ if $\mathcal{A}, A \not\models \lambda$ (for $\lambda = \mathtt{l}$ if $A \not\models \mathtt{l}$; for $\lambda = \mathsf{K}\,\mathtt{l}$ if $\mathcal{A} \not\models \mathsf{K}\,\mathtt{l}$).
- Thus, $\mathcal{A}$ is a *minimal model* of Π if

$$\mathcal{A}, A \models^{*} \Pi^{\langle \mathcal{A}, A \rangle} \text{ for every } A \in \mathcal{A}.$$

Let's see an example!

Example

Consider the following EASP program Γ:

$$\begin{aligned} p &\leftarrow \texttt{not}\, \sim q \\ \sim q &\leftarrow \texttt{not}\, p \\ r &\leftarrow \texttt{not}\, \mathrm{K}\, p \end{aligned}$$

Claim: $\mathcal{A} = \big\{\{p, r\}, \{\sim q, r\}\big\}$ is a minimal model of Γ: indeed,

$$\Gamma^{\{\underline{\{p,r\}},\{\sim q,r\}\}} : \begin{aligned} p &\leftarrow \top \\ \sim q &\leftarrow \bot \\ r &\leftarrow \top \end{aligned} \qquad \Gamma^{\{\{p,r\},\underline{\{\sim q,r\}}\}} : \begin{aligned} p &\leftarrow \bot \\ \sim q &\leftarrow \top \\ r &\leftarrow \top \end{aligned}$$

- $\big\{\underline{\{p, r\}}, \{\sim q, r\}\big\} \models \Gamma^{\{\underline{\{p,r\}},\{\sim q,r\}\}}$, but all its weakenings do not.
- $\big\{\{p, r\}, \underline{\{\sim q, r\}}\big\} \models \Gamma^{\{\{p,r\},\underline{\{\sim q,r\}}\}}$, but all its weakenings do not.

Comparison with K-WVs, SE-WVs, AEEMs and ESMs

where we differ? mainly for EASP programs including constraints...

EASP program Π	K-WVs	SE-WVs	AEEMs	ESMs
$\Pi_1 : p \text{ or } q$	$\{\{p\},\{q\}\}$	$\{\{p\},\{q\}\}$	$\{\{p\},\{q\}\}$	$\{\{p\},\{q\}\}$
$p \text{ or } q$ $\leftarrow \text{not } K\, p$	none	$\{\{p\}\}$	$\{\{p\}\}$	none
$\Pi_2 : p \leftarrow \text{not } q$ $q \leftarrow \text{not } p$ $r \leftarrow K\, p$	$\{\{p\},\{q\}\}$	$\{\{p\},\{q\}\}$	$\{\{p\},\{q\}\}$	$\{\{p\},\{q\}\}$
$p \leftarrow \text{not } q$ $q \leftarrow \text{not } p$ $r \leftarrow K\, p$ $\leftarrow \text{not } r$	$\{\{p, r\}\}$	$\{\{p, r\}\}$	$\{\{p, r\}\}$	none

Discussion: inclusion of belief operator

let's call our belief operator B :

1. can consider B as dual of K (same as M in ES), i.e.,
   - B is equivalent to `not` K `not`
   - can treat it neither positive nor negative construct (similar to `notnot` in ASP)
   - ? shoud we take its reduct? probably YES!
   - complicated because then we have to define how to take the reduct of K `not`
2. can consider B (similar to $\hat{K}$ in EEL) as non-dual of K
   - reasonable because EASP is a 3-valued formalism
   - treat it as a positive subjective literal like K p
   - and we do not take its reduct
   - ? but then $p \leftarrow B\,p$ has a unique ESM $\{\emptyset\}$. Intuitive?

Outline

To sum it up

- many different semantics approaches for ES
  - most of them are obsolete today:
    [Gelfond 1991,1994,2011; Kahl et al. 2014,2016, Wang&Zhang 2005,...]
  - successful candidates (to some extent):
    [Kahl 2018, SE 2016, FHS 2015]
    - cannot cope with programs including constraints
- Our approach:
  - propose a more standard generalisation of ASP
  - offer a solution to the constraint problem
  - still, it is not fully satisfactory.....

Future Work:

- improve knowledge-minimising property of ESMs
- better solution for programs including constraints
- make it more expressive inserting a belief operator

Thank you!