PEAR: a Tool for Reasoning About Scenarios and Probabilities in Description Logics of Typicality

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CILC 2019
Outline

- Extensions of DLs with Typicality and Probabilities:
  - Reasoning about ABox facts with probabilities of exceptions
  - PEAR: a reasoner for DL + T + probabilities
Description Logics

Important formalisms of knowledge representation

Two key advantages:
- well-defined semantics based on first-order logic
- good trade-off between expressivity and complexity
- at the base of languages for the semantic (e.g. OWL)

Knowledge bases

Two components:
- TBox: inclusion relations among concepts
- ABox: instances of concepts and roles, i.e. properties and relations among individuals
Description Logics

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**Knowledge bases**

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- **TBox**= inclusion relations among concepts
  - *Dog* ⊑ *Mammal*
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  - *Dog*(saki)
### Description Logics

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### Knowledge bases

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Description Logics

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Knowledge bases

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Extensions of DLs

DLs with nonmonotonic features

- need of representing prototypical properties and of reasoning about defeasible inheritance
- handle defeasible inheritance needs the integration of some kind of nonmonotonic reasoning mechanism
  - DLs + MKNF
  - DLs + circumscription
  - DLs + default
- all these methods present some difficulties ...
What are they?

- (with Laura Giordano, V. Gliozzi, N. Olivetti)
- Non-monotonic extensions of Description Logics for reasoning about prototypical properties and inheritance with exceptions
  - Basic idea: to extend DLs with a typicality operator $T$
  - $T(C)$ singles out the “most normal” instances of the concept $C$
  - semantics of $T$ defined by a set of postulates that are a restatement of Lehmann-Magidor axioms of rational logic $R$

Basic notions

- A KB comprises assertions $T(C) \sqsubseteq D$
- $T(Dog) \sqsubseteq Affectionate$ means “normally, dogs are affectionate”
- $T$ is nonmonotonic
  - $C \sqsubseteq D$ does not imply $T(C) \sqsubseteq T(D)$
## DLs with typicality

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The logic $\text{ALC} + T_{\text{min}}$

**Example**

$$ T(Pig) \sqsubseteq \neg \text{FireBreathing} $$

$$ T(Pig \sqcap Pokemon) \sqsubseteq \text{FireBreathing} $$

**Reasoning**

- ABox:
  
  $\text{Pig}(\text{tepig})$

- Expected conclusions:
  
  $\neg \text{FireBreathing}(\text{tepig})$
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**Reasoning**

- **ABox:**
  - $\text{Pig}(\text{tepig}), \text{Pokemon}(\text{tepig})$
- **Expected conclusions:**
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The logic $\mathcal{ALC} + T$

Semantics

- $\mathcal{M} = \langle \Delta^I, <, .^I \rangle$
  - additional ingredient: preference relation among domain elements
  - $<$ is an irreflexive, transitive, modular and well-founded relation over $\Delta^I$:
    - for all $S \subseteq \Delta^I$, for all $x \in S$, either $x \in Min_<(S)$ or $\exists y \in Min_<(S)$ such that $y < x$
    - $Min_<(S) = \{ u : u \in S \text{ and } \nexists z \in S \text{ s.t. } z < u \}$
  - Semantics of the $T$ operator: $(T(C))^I = Min_<(C^I)$
Weakness of monotonic semantics

Logic $\mathcal{ALC} + T$

- The operator $T$ is nonmonotonic, but...
- The logic is monotonic
  - If $KB \models F$, then $KB' \models F$ for all $KB' \supseteq KB$

Example

- In the KB of the previous slides:
  - if Pig(tepig) ∈ ABox, we are not able to
**Weakness of monotonic semantics**

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**Example**
- in the $KB$ of the previous slides:
  - if $Pig(tepig) \in ABox$, we are not able to:
    - assume that $T(Pig)(tepig)$
    - infer that $\neg FireBreathing(tepig)$
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The nonmonotonic logic $\text{ALC} + T_{\text{min}}$

### Rational closure

- Preference relation among models of a KB
  - $\mathcal{M}_1 < \mathcal{M}_2$ if $\mathcal{M}_1$ contains less exceptional (not minimal) elements
  - $\mathcal{M}$ minimal model of KB if there is no $\mathcal{M}'$ model of KB such that $\mathcal{M}' < \mathcal{M}$

- Minimal entailment
  - $\text{KB} \models_{\text{min}} F$ if $F$ holds in all minimal models of KB

- Nonmonotonic logic
  - $\text{KB} \models_{\text{min}} F$ does not imply $\text{KB}' \models_{\text{min}} F$ with $\text{KB}' \supset \text{KB}$

- Corresponds to a notion of rational closure of KB
Rational closure

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**Rational closure**

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DLs + T and probabilities

Introduction

- In the non-monotonic DL, all typicality assumptions that are consistent with the KB can be inferred.
- Counterintuitive, especially if we have hundreds of instances.
  - They are all typical dogs!!!!
DLs + T and probabilities

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**Introduction**

- $\mathcal{ALC} + T^P$: extension of $\mathcal{ALC}$ by typicality inclusions equipped by probabilities of exceptionality
- $T(C) \sqsubseteq_p D$, where $p \in (0, 1)$
- Intuitive meaning: normally, $C$s are $D$s, and the probability of having exceptional $C$s not being $D$s is $1 - p$

**Example**

$T(Student) \sqsubseteq_{0.3} SportLover$

$T(Student) \sqsubseteq_{0.9} SocialNetworkUser$

- Sport lovers and social network users are both typical properties of students
- Probability of not having exceptions is 30% and 90%, respectively
Basic idea

- extensions of an ABox containing only some of the “plausible” typicality assertions of the rational closure of KB
  - each extension represents a scenario having a specific probability
  - probability distribution among scenarios
  - nonmonotonic entailment restricted to extensions whose probabilities belong to a given and fixed range
  - reason about scenarios that are not necessarily the most probable
DLs + T and probabilities

**Extensions of ABox**

- typicality assumptions $\mathbf{T}(C_1)(a_1), \mathbf{T}(C_2)(a_2), \ldots, \mathbf{T}(C_n)(a_n)$ inferred from $\mathcal{ALC} + \mathbf{T}_{min}$
- extensions of ABox obtained by choosing some typicality assumptions
  - $\widetilde{\mathcal{A}}_1 = \{ \mathbf{T}(C_1)(a_1), \mathbf{T}(C_2)(a_2), \ldots, \mathbf{T}(C_n)(a_n) \}$
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- reasoning in the monotonic $\mathcal{ALC} + T$ considering TBox and ABox extended with chosen assumptions
DLs + T and probabilities

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- reasoning in the monotonic $\mathcal{ALC} + T$ considering TBox and ABox extended with chosen assumptions
Extensions of ABox and probabilities

\[ T(C) \sqsubseteq 0.3 \quad D \]
\[ T(C) \sqsubseteq 0.7 \quad E \]
\[ T(F) \sqsubseteq 0.8 \quad G \]
\[ T(C) \sqsubseteq 0.5 \quad H \]
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\end{align*}
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\[0.3 \times 0.7 \times 0.5\]
Extensions of ABox and probabilities

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\end{align*}
\]

\[
\begin{align*}
T(C)'(a) & \quad T(F)(a) & \quad T(C)(b) \\
0.3 \times 0.7 \times 0.5 & = 0.105
\end{align*}
\]
Extensions of ABox and probabilities

\[ T(C) \subseteq_{0.3} D \]
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\[ T(C)(a) \quad T(F)(a) \quad T(C)(b) \]

\[ 0.105 \]
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\[ T(F) \subseteq 0.8 \quad G \]
\[ T(C) \subseteq 0.5 \quad H \]

\[ T(C) \quad (a) \quad T(F) \quad (a) \quad T(C) \quad (b) \]

\[ 0.3 \times 0.7 \times 0.5 \]

\[ 0.105 \quad 0.8 \]
Extensions of ABox and probabilities

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\begin{align*}
T(C) \subseteq_0.3 & \quad D \\
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\end{align*}
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\[
\begin{align*}
0.3 \times 0.7 \times 0.5 & \quad 0.105 & \quad 0.8 & \quad 0.105
\end{align*}
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### Extensions of ABox and probabilities

<table>
<thead>
<tr>
<th>Condition</th>
<th>Probability</th>
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- $T(C)(a)$: 0.105
- $T(F)(a)$: 0.8
- $T(C)(b)$: 0.105

- $0.3 \times 0.7 \times 0.5$
Extensions of ABox and probabilities

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0.3 \times 0.7 \times 0.5 & \\
\end{align*}

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\mathbf{A}_1 & \quad \mathbf{T}(C)(a) & \quad \mathbf{T}(F)(a) & \quad \mathbf{T}(C)(b) \\
[0.105, 0.8, 0.105] & \\
\end{align*}
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Extensions of ABox and probabilities

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\end{align*}
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\begin{array}{c|ccc}
 & \text{T}(C)(a) & \text{T}(F)(a) & \text{T}(C)(b) \\
\hline
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0.3 \times 0.7 \times 0.5 & \end{array}
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\tilde{A}_2
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- \(\bar{A}_1\) \(\bar{A}_2\) \(\bar{A}_3\) \(\vdots\) \(\bar{A}_8\)
  - \([0.105, 0.8, 0.105]\)
  - \([0.105, 0, 0]\)
  - \([0, 0.8, 0.105]\)
  - \([0, 0, 0]\)
Extensions of ABox and probabilities

\[
\begin{align*}
\mathbb{T}(C) &\subseteq 0.3\ D \\
\mathbb{T}(C) &\subseteq 0.7\ E \\
\mathbb{T}(F) &\subseteq 0.8\ G \\
\mathbb{T}(C) &\subseteq 0.5\ H
\end{align*}
\]

\[
\begin{array}{cccc}
\mathbb{T}(C)(a) & \mathbb{T}(F)(a) & \mathbb{T}(C)(b) \\
0.105 & 0.8 & 0.105 \\
0.3 \times 0.7 \times 0.5
\end{array}
\]

\[
\begin{align*}
\mathbb{P}\mathbf{A}_1 &= 0.105 \times 0.8 \times 0.105 \\
\mathbb{P}\mathbf{A}_2 &= 0.105 \times (1 - 0.8) \times (1 - 0.105) \\
\mathbb{P}\mathbf{A}_3 &= (1 - 0.105) \times 0.8 \times 0.105 \\
\mathbb{P}\mathbf{A}_8 &= (1 - 0.105) \times (1 - 0.8) \times (1 - 0.105)
\end{align*}
\]
Entailment

- Given $KB = (\mathcal{T}, \mathcal{A})$ and $p, q \in (0, 1]$
- $\mathcal{E} = \{\widehat{A_1}, \widehat{A_2}, \ldots, \widehat{A_k}\}$ set of extensions of $\mathcal{A}$ whose probabilities are $p \leq P_1 \leq q, p \leq P_2 \leq q, \ldots, p \leq P_k \leq q$
- $\mathcal{T}' = \{\mathcal{T}(C) \sqsubseteq D \mid \mathcal{T}(C) \sqsubseteq r, D \in \mathcal{T}\} \cup \{C \sqsubseteq D \in \mathcal{T}\}$
- $KB \models_{\mathcal{ALC}+\mathcal{T}P} F$
  - if $F$ is $C \sqsubseteq D$ or $\mathcal{T}(C) \sqsubseteq D$, if $(\mathcal{T}', \mathcal{A}) \models_{\mathcal{ALC}+\mathcal{T_{min}}} F$
  - if $F$ is $C(a)$, if $(\mathcal{T}', \mathcal{A} \cup \widehat{A_i}) \models_{\mathcal{ALC}+\mathcal{T}} F$ for all $\widehat{A_i} \in \mathcal{E}$
- probability of $F$: $\mathbb{P}(F) = \sum_{i=1}^{k} P_i$
DLs $+ \ T$ and probabilities

**Entailment**

- Given $KB = (T, A)$ and $p, q \in (0, 1]$
- $E = \{\overset{\sim}{A}_1, \overset{\sim}{A}_2, \ldots, \overset{\sim}{A}_k\}$ set of extensions of $A$ whose probabilities are $p \leq P_1 \leq q, p \leq P_2 \leq q, \ldots, p \leq P_k \leq q$
- $T' = \{T(C) \sqsubseteq D \mid T(C) \sqsubseteq rD \in T\} \cup \{C \sqsubseteq D \in T\}$
- $KB \models_{\mathcal{ALC} + T^p} F$
  - if $F$ is $C \sqsubseteq D$ or $T(C) \sqsubseteq D$, if $(T', A) \models_{\mathcal{ALC} + T_{\text{min}}} F$
  - if $F$ is $C(a)$, if $(T', A \cup \overset{\sim}{A}_i) \models_{\mathcal{ALC} + T} F$ for all $\overset{\sim}{A}_i \in E$
- Probability of $F$: $\mathbb{P}(F) = \sum_{i=1}^{k} P_i$
**Entailment**

- Given $\text{KB} = (\mathcal{T}, \mathcal{A})$ and $p, q \in (0, 1]$
- $\mathcal{E} = \{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_k\}$ set of extensions of $\mathcal{A}$ whose probabilities are $p \leq P_1 \leq q, p \leq P_2 \leq q, \ldots, p \leq P_k \leq q$
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- Probability of $F$: $P(F) = \sum_{i=1}^{k} P_i$
DLs + T and probabilities

Entailment

- Given \( KB = (T, A) \) and \( p, q \in (0, 1] \)
- \( E = \{ \overline{A_1}, \overline{A_2}, \ldots, \overline{A_k} \} \) set of extensions of \( A \) whose probabilities are \( p \leq P_1 \leq q, p \leq P_2 \leq q, \ldots, p \leq P_k \leq q \)
- \( T' = \{ T(C) \sqsubseteq D \mid T(C) \sqsubseteq_r D \in T \} \cup \{ C \sqsubseteq D \in T \} \)
- \( KB \models_{\mathcal{ALC} + TP} F \)
  - if \( F \) is \( C \sqsubseteq D \) or \( T(C) \sqsubseteq D \), if \( (T', A) \models_{\mathcal{ALC} + T_{\text{min}}} F \)
  - if \( F \) is \( C(a) \), if \( (T', A \cup \overline{A_i}) \models_{\mathcal{ALC} + T} F \) for all \( \overline{A_i} \in E \)
- probability of \( F \): \( P(F) = \sum_{i=1}^{k} P_i \)
DLs + T and probabilities

### Entailment

- Given $KB = (T, A)$ and $p, q \in (0, 1]$.
- $E = \{\overline{A_1}, \overline{A_2}, \ldots, \overline{A_k}\}$ set of extensions of $A$ whose probabilities are $p \leq P_1 \leq q, p \leq P_2 \leq q, \ldots, p \leq P_k \leq q$.
- $T' = \{T(C) \sqsubseteq D \mid T(C) \sqsubseteq r, D \in T\} \cup \{C \sqsubseteq D \in T\}$.
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  - if $F$ is $C \sqsubseteq D$ or $T(C) \sqsubseteq D$, if $(T', A) \models_{\mathcal{ALC}+T_{\min}} F$.
  - if $F$ is $C(a)$, if $(T', A \cup \overline{A_i}) \models_{\mathcal{ALC}+T} F$ for all $\overline{A_i} \in E$.
- Probability of $F$: $P(F) = \sum_{i=1}^{k} P_i$. 

Gian Luca Pozzato
DLs + T and probabilities

**TBox**

- PokemonCardPlayer ⊑ CardPlayer
- $T(\text{CardPlayer}) \sqsubseteq_{0.85} \neg \text{YoungPerson}$
- $T(\text{PokemonCardPlayer}) \sqsubseteq_{0.7} \text{YoungPerson}$
- $T(\text{Student}) \sqsubseteq_{0.6} \text{YoungPerson}$
- $T(\text{Student}) \sqsubseteq_{0.8} \text{InstagramUser}$

**Inferences**

- $T(\text{CardPlayer} \sqcap \text{Italian}) \sqsubseteq \neg \text{YoungPerson}$ is entailed in $\mathcal{ALC} + T^P$
- if $\mathcal{A} = \{\text{PokemonCardPlayer}(lollo), \text{Student}(lollo), \text{Student}(poz)\}$:
  - YoungPerson(lollo) has probability 70%
  - InstagramUser(poz) has probability 48%
DLs + T and probabilities

TBox

PokemonCardPlayer ⊑ CardPlayer

\[ T(\text{CardPlayer}) \sqsubseteq_{0.85} \neg \text{YoungPerson} \]

\[ T(\text{PokemonCardPlayer}) \sqsubseteq_{0.7} \text{YoungPerson} \]

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\[ T(\text{Student}) \sqsubseteq_{0.8} \text{InstagramUser} \]

Inferences

- \[ T(\text{CardPlayer} \sqcap \text{Italian}) \sqsubseteq \neg \text{YoungPerson} \] is entailed in \[ \mathcal{ALC} + T^P \]

- if \[ \mathcal{A} = \{ \text{PokemonCardPlayer}(lollo), \text{Student}(lollo), \text{Student}(poz) \} \]:
  - \[ \text{YoungPerson}(lollo) \] has probability 70%
  - \[ \text{InstagramUser}(poz) \] has probability 48%
DLs + T and probabilities

New results

- New results in two directions:
  - with Antonio Lieto: semantics
    - probability as proportion:
      \[
      \frac{|\{x \in C^\mathcal{I} \mid x \notin (T(C))^\mathcal{I} \text{ and } x \in (\neg D)^\mathcal{I}\}|}{|C^\mathcal{I}|} \leq 1 - p
      \]
    - probability as degree of belief: distributed semantics inspired by DISPONTE (Bellodi, Cota, Riguzzi, Zese)
  - submitted at the special issue of CILC 2018
Probability of Exceptions and Typicality Reasoner

- Python implementation of the reasoning services provided by the logic $\mathcal{ALC} + T^p$
- Makes use of owlready2 to rely on HermiT
- Exploits the translation into standard $\mathcal{ALC}$
- Available at http://di.unito.it/pear
Basic ideas

- compute extensions $\mathcal{A}$ of the ABox and corresponding alternative scenarios $\tilde{\mathcal{A}}_i$
  - very expensive...optimizations needed
- compute probabilities of each scenario
- select the extensions whose probabilities belong to a given range $\langle p, q \rangle$
- check whether a query $F$ is entailed from all the selected extensions in the monotonic logic $\mathcal{ALC} + T$
PEAR relies on a polynomial encoding of $\mathcal{ALC} + \mathbf{T}$ into $\mathcal{ALC}$ (Giordano, Gliozzi, Olivetti)

- exploits the definition of $\mathbf{T}$ in terms of a Gödel-Löb modality $\Box$:
  - $\mathbf{T}(C)$ defined as $C \sqcap \Box \neg C$ where the accessibility relation of $\Box$ is the preference relation $<$ in $\mathcal{ALC} + \mathbf{T}$ models
Beyond COCOS

Future works

- Reasoning in real application domains:
  - which range of probabilities?
- Extension to other DLs
- Learning of a knowledge base with DLs + $T$ + probabilities
- Optimization of PEAR by the application of techniques introduced by (Alberti, Bellodi, Cota, Lamma, Riguzzi, Zese)
Any question?