FO queries strongly distributing over components

F. Di Cosmo

CILC
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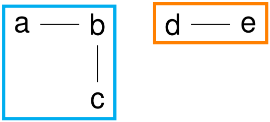

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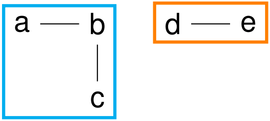

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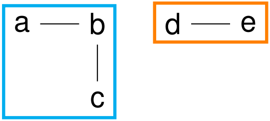

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Definition

Two facts are **directly connected**, $\sim_d$, if they share some individual.
The **connection relation**, $\sim$, is the transitive closure of $\sim_d$.
The **set of connected components** of D, **cc**(D), is the set of equivalence classes of $\sim$.

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Definition

A query q **distributes** if, for each database D:

$$q(D) = \bigcup_{C \in \mathbf{CC}(D)} q(C)$$

Main question

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A query distributes iff it has a connected specification.

$E(x,z) \leftarrow R(x,y), P(z,w), \neg R(x,z)$ is not connected.

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Structures: Databases $\rightsquigarrow$ FO relational structures.

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- Monotony: $S' \subset S \Rightarrow q(S') \subset q(S)$
- Locality: $\mathbf{a} \in q(S) \Rightarrow \exists C \in \text{cc}(S)\ \mathbf{a} \in q(C)$

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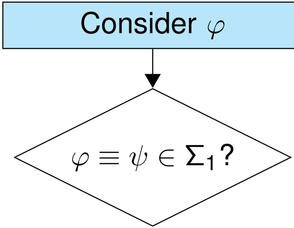

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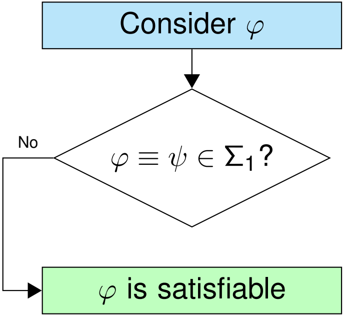

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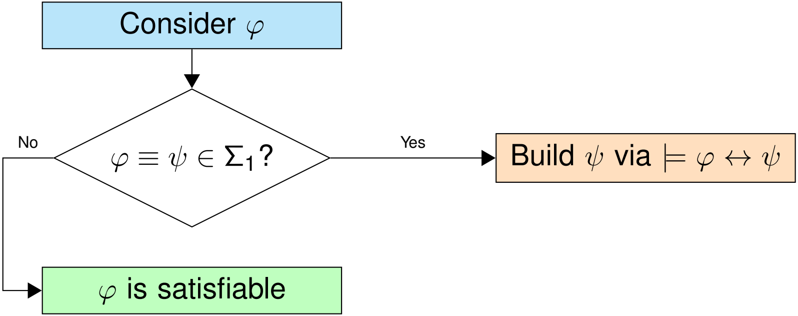

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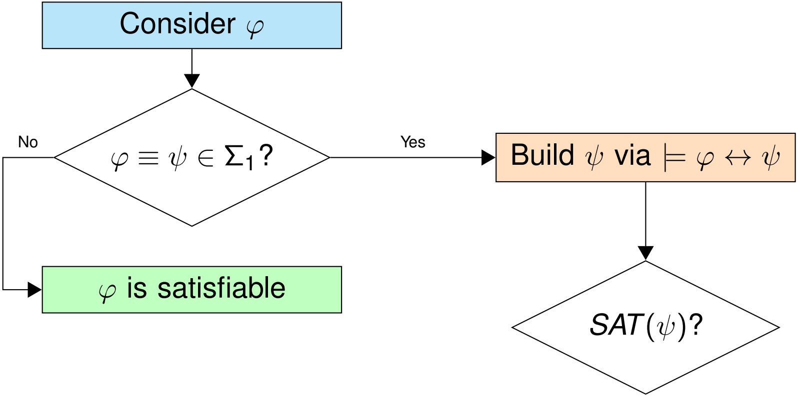

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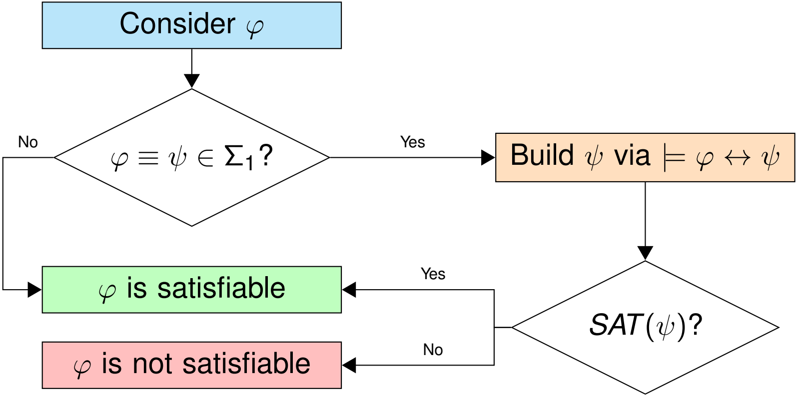

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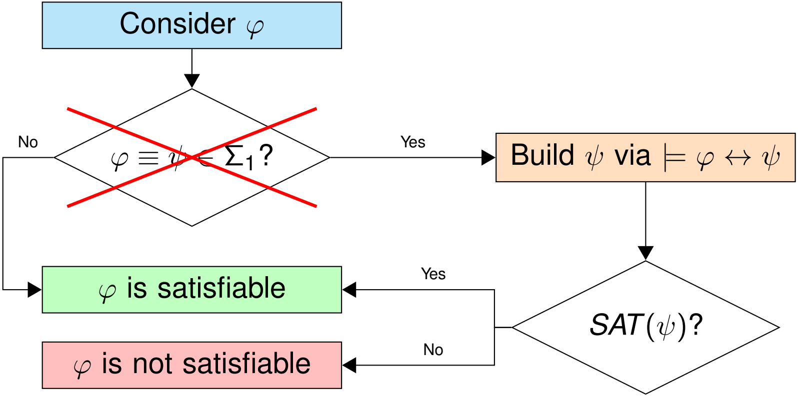

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A $\bigwedge_{i \in I} L_i$ *is local iff:*

- *it is contradictory or*
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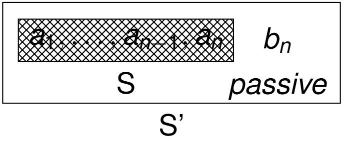
$\varphi(x_1, \ldots, x_n)$

x_n *only in* $\neg A$



Conclusions

- Undecidability.
- Safeness (*locality*).
- Strong distribution vs *Simple* distribution.
- Constraints on structures.

Essential bibliography

Ameelot, Tom J. et al.
Datalog queries distributing over components
ACM TOCL, 2017.

Berger, Gerald and Pieris, Andreas
Ontology mediated queries distributing over components
IJCAI, 2016.