Model Completeness, Covers and Superposition

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June 19, 2019

Outline

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- Thanks to the presence of data, the resulting models are *intrinsically* infinite-state.

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- Gulwani and Musuvathi [ESOP, 2008] introduced the notion of a ***cover***, which provides precise computation of reachable states.
- They showed that covers exist for **EUF** and proved that its computation becomes *tractable* with only unary free function symbols.

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- We prove that ***computing covers*** for a theory is **equivalent** to ***eliminating quantifiers*** in its model completion.
- We show that covers for **EUF** can be computed through a constrained version of the **Superposition Calculus**, equipped with appropriate settings and reduction strategies.

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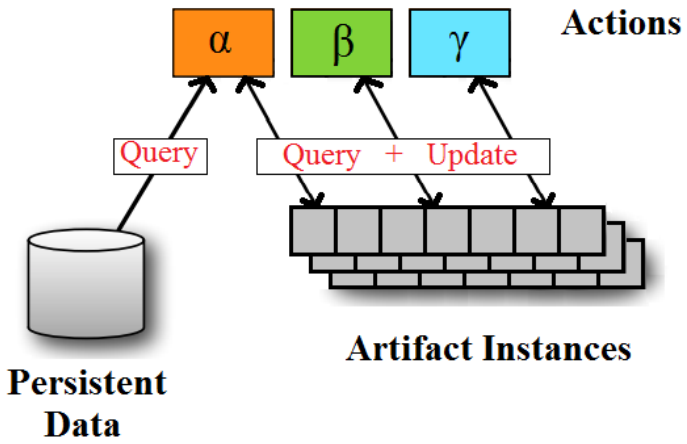

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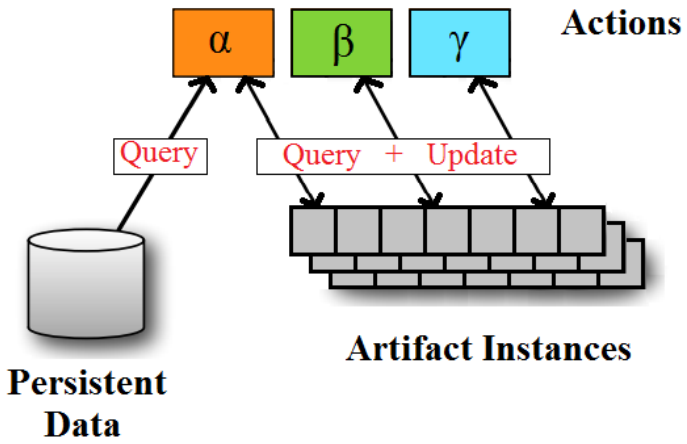

We formalize Artifact-Centric Systems in an array-based formal model. This encoding allows us to exploit the powerful machinery provided by SMT-techniques.

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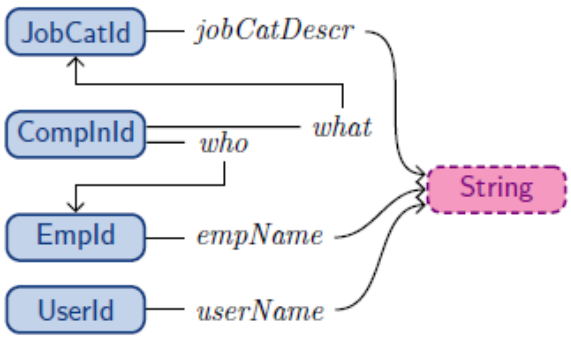

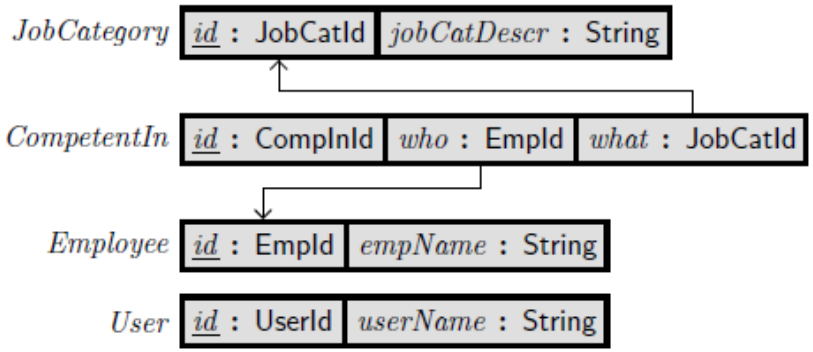

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where (Σ, T) is a **(read-only) DB schema**, $\underline{x}$ are *individual* variables (for **artifact variables**, i.e. the working memory).
The Σ-formula ι represents the ***initialization*** of $\mathcal{S}$, whereas the Σ-formula τ models the ***transitions*** (actions) of $\mathcal{S}$.

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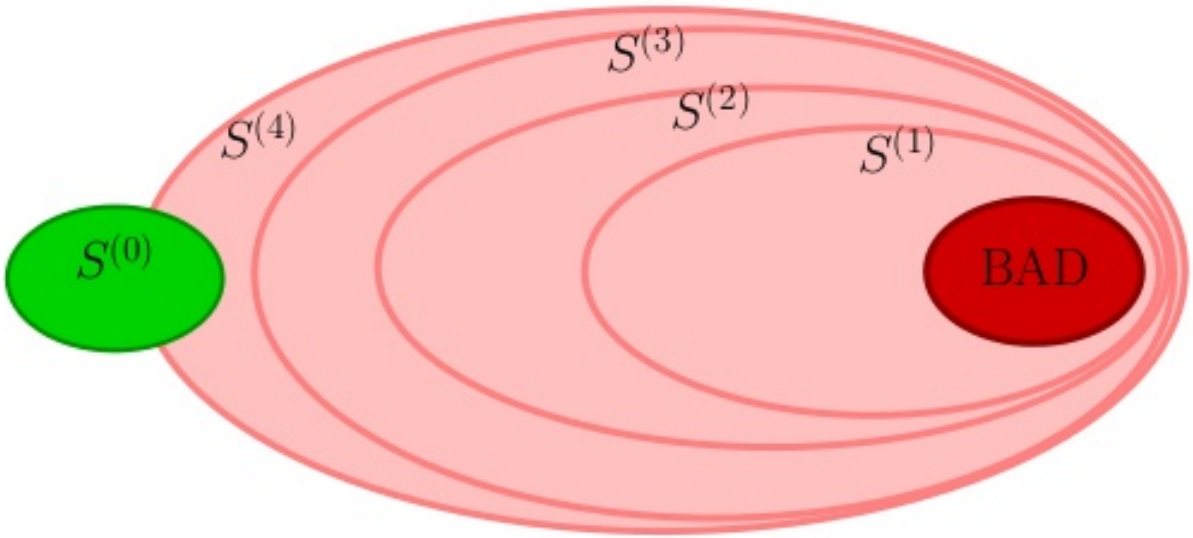
Backward Reachability: intuition

Backward-Reachability ($S^{(0)} \equiv$ "bad states")

Safety Check	If $S^{(i)}$ contains an initial, return **unsafe**
Next States	Compute $S^{(i+1)} := S^{(i)} \cup T^{-1}(S^{(i)})$
Fix-Point Check	If $S^{(i+1)} \equiv S^{(i)}$, return **safe**



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The proof requires **Quantifier Elimination**!

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Remark

*Detecting **unsafe conditions** in T or in $T^\star$ are equivalent problems!*

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Fix a theory T and an existential formula $\exists \underline{e}\, \phi(\underline{e}, \underline{y})$.

- We call a *residue* of $\exists \underline{e}\, \phi(\underline{e}, \underline{y})$ any quantifier-free formula in $Res(\exists \underline{e}\, \phi) = \{\theta(\underline{y}, \underline{z}) \mid T \models \phi(\underline{e}, \underline{y}) \rightarrow \theta(\underline{y}, \underline{z})\}$. A quantifier-free formula $\psi(\underline{y})$ is a T-***cover*** of $\exists \underline{e}\, \phi(\underline{e}, \underline{y})$ iff $\psi(\underline{y}) \in Res(\exists \underline{e}\, \phi)$ and $\psi(\underline{y})$ implies (modulo T) all the other formulae in $Res(\exists \underline{e}\, \phi)$.

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Theorem

Suppose that T is a universal theory. Then, T has a model completion T^* ***iff*** *T has uniform quantifier-free interpolation. If this happens, T^* is axiomatized by the infinitely many sentences $\forall \underline{y}\, (\psi(\underline{y}) \rightarrow \exists \underline{e}\, \phi(\underline{e}, \underline{y}))$, where $\exists \underline{e}\, \phi(\underline{e}, \underline{y})$ is a primitive formula and ψ is a* ***cover*** *of it.*

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Remark: termination requires a precise ***application strategy*** for the rules.

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This also proves the existence of the model completion of EUF. Notice also that EUF suffices to represent DB theories of our Artifact Systems

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- We computed covers for **EUF** adopting a **constrained version of the Superposition Calculus** with appropriate application strategies.

THANKS FOR YOUR ATTENTION!